Review for Quiz 1

L1, L2 \to regular

M1, M2 \to DFA \quad L(M1) = L1, \quad L(M2) = L2

Union ($\cup$): $M_i$

Concatenation: $L(M) = L(M1) \cup L(M2)$

Kleene Star(*):
Complement of $L_1$:
M: Change all the final states in $M_1$ to non-final, and all the normal states to final.

Intersection:

$$L_1 \cap L_2 = (L_1' \cap L_2')$$

Difference:

$$L_1 - L_2 = L_1 \cap L_2'$$

SAMPLE QUIZ

#4) $L$ is regular

$L' = \{xy \mid x \in L \text{ and } y \in \overline{L}\}$

Is $L'$ regular?

Ans: $L'$ is regular

Proof

$L' = L \cdot \overline{L}$

Since, $L$ is regular, and regular languages are closed under complement and concatenation, $\overline{L}$ is regular, and so is $L \overline{L}$, thus $L'$ is regular.

#5) Let $L = \{0^m1^{2m} \mid m > 0\}$ show $L$ is not regular.

Proof: Assume that $L$ is regular. Then by Pumping Lemma, $\exists$ positive integer $n$,

such that $\forall w \in L$, if $|w| \geq n$, then $\exists x, y, z$ such that $w = xyz$, $|xy| \leq n$, $y \neq e$, and $\forall i \geq 0$: $xy^i \in L$.

Now select a string a particular string $w_0 = 0^n1^{2n} \in L$. Since $|w_0| = n + 2^n$

$n$, $\exists x, y, z$ such that $w_0 = xyz$, $|xy| \leq n$, $y \neq e$, and $\forall i \geq 0$: $xy^i \in L$. 

Since \(|xy| \leq n\), \(xy\) must be a prefix of \(0^n\). Since \(y \neq e\), \(|y| > 0\). Hence, \(n - |y| < n\).

Thus, \(xy^n z = 0^n y^n 1^n \in L\), which is a contradiction. Therefore, \(L\) is not regular.

#1) DFA

\[ (Q, \Sigma, \vartheta, S, F) \]

Formal Definition

\[
Q = \{ q_0, q_1, q_2, q_3 \} \\
\Sigma = \{ a, b \} \\
S = q_0 \\
F = \{ q_2 \} \\
\]

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