Class Notes:
TA Office Hours have changed to Tuesdays 10:00-12:00 in OS219.

Homework 3 Review:

2.19 a)

NFA Construction Review:

R1R2:

R1*

R1 U R2
Convert: \((a \cup b)^* ab(abb \cup a^*)^* bb^*\) to NFA

R1: \((a \cup b)^*\)

R2: \(ab\)

R3: \((abb \cup a^*)^*\)
R4: $bb^*$

Completed NFA:

2.22 c)

Convert DFA to Regular Expression.

DFA:
Eliminate State 1:

Eliminate State 2:

Eliminate State 3:

Eliminate State 4:
Lecture Notes:

Known:
- Given a regular expression, there is an algorithm to generate an NFA accepting it.
- There is an Algorithm that generates a regular expression from a Finite Automata.
- There is an Algorithm to convert NFA’s to DFA’s.
  - The DFA generated may have more states than necessary. There might be a different DFA accepting the same Language but with a smaller number of states. Such a DFA is more desirable.

How do we find the “best” DFA for the same regular expression? (“Best” means the DFA has the smallest number of states among all DFA’s that accept the same regular expression.)

To find the “Best” DFA we need to identify equivalent states. Then these equivalent states can be replaced by a single new state.

Q: What does it mean for two states p & q to be equivalent?
A:

We can identify equivalent states by using the following algorithm:

0. Remove all unreachable states (i.e. if a state is not reachable from s, then remove it).

1. List all unordered state pairs (p,q), where p ∈ F and q ∉ F.
   - Let U be the set of these pairs. (we know that (p,q) ∈ U then p & q are not equivalent.)

2. For each pair (p,q) ∈ U we want to know whether p & q are equivalent. If δ(p,a) = p', δ(q,a) = q' then draw (p,q) →^a (p',q').
   - If (p',q') ∈ U , then p & q are not equivalent.
   - If p' = q', then this path ends.
   - If (p', q') occurs earlier in the path, then this path stops.

3. Repeat step 2 for every symbol a and for all new state pairs.
This algorithm will stop because $Q \times Q$ is finite. When it stops, if there is a path from $(p, q)$ to a state in $U$, then $p$ & $q$ are not equivalent. Otherwise, $p$ & $q$ are equivalent.

4. Assume $p$, $q$, $r$, … are equivalent to each other, then make them a single state and the resulting DFA is the minimum DFA.

Example:

There are no unreachable states in this DFA. $U$: \{(s,t),(s,p),(q,t),(q,p),(r,t),(r,p),(m,t),(m,p)\}$

- $(s,q) \xrightarrow{a} (q,t) \in U$, Hence, $s$ & $q$ are not equivalent.
- $(s,r) \xrightarrow{a} (q,p) \in U$, Hence, $s$ & $r$ are not equivalent.
- $(s,m) \xrightarrow{a} (q,m) \xrightarrow{a} (t,m) \in U$, Hence $s$ & $m$, $q$ & $m$ are not equivalent.
- $(s,m) \xrightarrow{b} (r,m) \in U$
- $(r,m) \xrightarrow{a} (p,m) \in U$, Hence, $r$ & $m$ are not equivalent.
- $(r,q) \xrightarrow{a} (p,t) \in U$
- $(r,q) \xrightarrow{b} (m,m) \in U$, Because $p$ & $t$ are equivalent, $m$ & $m$ are equivalent.

$r$ & $q$ are equivalent.