Homework Set #2

Assigned: Tuesday, 2/3 Due: Tuesday, 2/10 (start of lecture)

This assignment covers textbook material in Chapters 1-3.
Note: Refer to course web site for homework policies.
Remember to attach signed honor statement.

1. (28 points) Function Order of Growth and $O, \Omega, \Theta$ Notation Practice:

(a) (12 points) List the 4 functions below in nondecreasing asymptotic order of growth:

\[
\begin{align*}
\lg^2 n & \quad n^{(\lg n)} & \quad n^{(\lg(1/2))} & \quad 3^{\log_3(n + 1/n)} \\
\text{1) smallest} & \quad \text{2) } & \quad \text{3) } & \quad \text{4) largest}
\end{align*}
\]

Justify your answer mathematically by showing values of $c$ and $n_0$ for each pair of functions that are adjacent in your ordering.

Given (for large $n$):

1) $f_1(n) \in \Omega \left(3^{\log_3(n + 1/n)}\right)$
2) $f_2(n) \in O(\lg^2 n)$
3) $f_3(n) \in \Omega(1/n)$
4) $f_4(n) \in \Theta(n^{(\lg n)})$

Circle TRUE or FALSE for each statement below. Circle TRUE if the statement must always be true, given the assumptions; FALSE otherwise. In the TRUE case, provide a proof. In the FALSE case, give a counter-example.

(b) (4 points) $f_2(n) \in O(f_1(n))$ TRUE FALSE

(c) (4 points) $f_4(n) \in \Omega(f_2(n))$ TRUE FALSE

(d) (4 points) $f_3(n) \in O(f_4(n))$ TRUE FALSE

(e) (4 points) $f_1(n) \in \Theta(f_3(n))$ TRUE FALSE
2. (10 points) **More \(O, \Omega, \Theta\ Notation Practice:** Textbook Exercise 3.1-1 on p. 52. Justify your answer.

3. (15 points) **More \(O, \Omega, \Theta\ Notation Practice:** Textbook Exercise 3.2-8 on p. 60. Justify your answer.

4. (15 points) **More \(O, \Omega, \Theta\ Notation Practice:** Textbook Problem 3-4 part (g) on p. 62. Justify your answer.

5. (32 points) **Pseudocode Analysis:** For the pseudocode below for \(\text{Mystery}(n)\), derive a tight upper bound on its asymptotic \textit{worst-case} running time \(T(n)\). That is, for the set of inputs including those that force \(\text{Mystery}\) to work its hardest, find \(g(n)\) such that \(T(n) \in O(g(n))\). Assume that the input \(n\) is a positive integer \(\geq 4\) that is divisible by 4. Justify your answer.

\[
\begin{align*}
\text{Mystery}(n) \\
1 & \quad \text{for } i = 1 \text{ to } n \\
2 & \quad \quad \text{if } i == 4 \\
3 & \quad \quad \quad \text{for } j = 1 \text{ to } i \\
4 & \quad \quad \quad \quad \text{print "Mystery Case 1"} \\
5 & \quad \quad \text{for } k = 1 \text{ to } n/4 \\
6 & \quad \quad \text{print "Mystery Case 2"}
\end{align*}
\]