Homework Set #1

Assigned: Tuesday, 1/20  Due: Thursday, 1/29 (start of lecture)

This assignment covers textbook material in Chapters 1-2.

Note: Refer to course web site for homework policies.

Remember to attach signed honor statement.

1. (10 points) Comparing Functions: What is the smallest integer value of $n > 2$ such that an algorithm whose running time is $1500 \log_{10} n$ (that is: $1500(\log_{10} n) \log n$) runs faster than an algorithm whose running time is $n^2 \log_{10} n$ on the same machine? Justify your answer.

2. (10 points) Sorting: Using Figure 2.2 on p. 18 of our textbook as a model, illustrate the operation of `INSERTION-SORT` on the array $A = < 28, 7, 93, 56>$.

3. (10 points) Sorting: Using Figure 2.4 on p. 35 of our textbook as a model, illustrate the operation of `MERGE-SORT` on the array $A = < 28, 7, 93, 56>$.

4. (20 points) Analysis: Consider the following pseudocode procedure for an array $A$ of $n$ integers:

```plaintext
1 MYSTERY(n, A)
2   for i = 1 to n^2
3      for j = 1 to i
4         INSERTION-SORT(A)
5         print “Welcome to sorting!”
```

Derive the tightest upper bound on the worst-case asymptotic running time of `MYSTERY` that you can. Assume that $n$ is a positive integer.

5. (20 points) `MYSTERY(n)`

```plaintext
1   if n ≤ 1
2      return 1
3   for i = 1 to 5
4      for j = 1 to n^2
5         print “Welcome to recursion!”
6   MYSTERY(n/3)
7   MYSTERY(n/3)
8   MYSTERY(n/3)
```

Analyze the worst-case asymptotic execution time of `MYSTERY`. Express the execution time as a function of the input value $n$. Assume that $n = 3^k$ for some positive integer $k \geq 1$. 
6. (30 points) **Algorithm Design** (Goodrich & Tamassia): Suppose that you are given an array $A$ of $n$ elements, each of which is colored red or blue. Assume that red and blue are represented using ‘R’ and ‘B’ characters, respectively, in array $A$. Give an efficient in-place method for ordering $A$ so that all the blue elements are listed before all the red elements. Provide pseudocode using the conventions in our textbook. Justify both “mechanical” and “as advertised” correctness of your pseudocode. Analyze an upper bound on the worst-case asymptotic running time of your algorithm.