Homework Set #1

Assigned: Wednesday, 9/3  
Due: Monday, 9/8 (start of lecture)

This assignment covers textbook material in Chapter 0.  
Note: Refer to course web site for homework policies.  
Remember to attach signed honor statement.

1. (15 points) Set Descriptions:
   a) Create a set $A$ of sets where $|A| = 3$ and this statement holds:
      $$(\forall x \in A)(\exists y \in A) \quad x \neq y \quad \text{and} \quad \left( |x| = |y| \Rightarrow \prod_{i \in x} i = \prod_{j \in y} j \right)$$

   b) Give an example of a set of sets that violates the statement in (a).

   c) Let $Z_n^* = \{[a]_n \in Z_n | \gcd(a, n) = 1\}$
      where $[a]_n$ is an equivalence class modulo $n$ (i.e. $[a]_n = \{a + kn | \quad k \in Z\}$),
      and $Z_n$ is the set of all such equivalence classes modulo $n$
      (i.e. $Z_n = \{[a]_n | \quad 0 \leq a \leq n-1\}$).
      As an example, $Z_6^* = \{[1]_6, [5]_6\}$
      I. List the elements of $Z_{10}^*$.
      II. Write a short, informal English description of $Z_n^*$.

2. (10 points) Propositional Logic: Create a truth table for the following logical expression:
   $$((P \rightarrow Q) \lor (Q \rightarrow P)) \land (\neg Q)$$

3. (15 points) Equivalence Relation: Let $S$ be a finite set, and let $R$ be an equivalence relation on $S$ such that the domain of $R$ is $S \times S$. Prove that if $R$ is also anti-symmetric, then the equivalence classes of $S$ with respect to $R$ are singletons.

4. (60 points) Practicing Types of Proofs:
   a) Prove that if $s$ and $t$ are rational numbers and $t \neq 0$, then $s/t$ is a rational number.

   b) Prove, by mathematical induction on $n$, that, for every natural number:
      $$1(1!) + \ldots + n(n!) = (n+1)! - 1$$
      That is, show that: $$\sum_{i=1}^{n} i(i!) = (n+1)! - 1.$$