Congestion Control with QoS and Delays Utility Function

Hengky Susanto and Byung Guk Kim

Abstract — This paper primarily addresses how available bandwidth should be optimally distributed among competing streams of elastic traffic like TCP traffic while taking Quality-of-Service (QoS) and delay into consideration. Network Utility Maximization (NUM) in [1], a congestion control algorithm, allows users to set for an optimum network-wide rate allocation through their utility. By incorporating delay into utility function, users can accommodate for QoS requirements.

Index Terms — About networks, congestion control, resource management, optimization methods, QoS.

I. INTRODUCTION

With the increasing demand for bandwidth along with the increasing size of data network transmission, the network becomes overburdened and user may experience connection quality degradation. The mismanagement of bandwidth allocation may lead to bottleneck where the amount of data that is transmitted into the network exceeds the capacity. If the demands exceed the capacity, performance is generally poor and unpredictable. Thus, an appropriate model for bandwidth allocation becomes an important task in assuring high network performance. Network bandwidth allocation was formulated as a Network Utility Maximization (NUM) problem by F. Kelly [1][2]. The NUM formulation attempts to maximize the aggregate utility of users receiving bandwidth subject to limits on the link capacity,

\[
\text{maximize } \sum_{s \in S} U_s(x_s)
\]

s.t. \(Ax \leq CmHy = x\)

over \(x, y \geq 0\)

Here, \(C\) denotes a set of capacity of link \(l\), for \(l \in L\), where \(L\) denotes a set of links in the network and \(S\), a set of users accessing the network. The matrix \(A\) has the routing information that link \(l\) is associated with route \(r\) and matrix \(R\) has the path of user \(s\), such that \(A = (A_{lr}, l \in L, r \in R)\), where \(A_{lr} = 1\) if \(l \in r\), and \(A_{lr} = 0\), otherwise. Let \(H_{sr} = 1\) if path \(r\) is associated with users, and \(H_{sr} = 0\), otherwise resulting in the matrix \(H = (H_{sr}, s \in S, r \in R)\). Variable \(y\) is a set of flow traverse over router. In addition, the utility function is assumed to be non-decreasing smooth function, strictly concave, and differentiable in \(x > 0\). These conditions are necessary for convex optimization [2]. Kelly has demonstrated that network traffic flows can be regulated in a proportionally fair manner with a distributed approach [1].

Kelly’s framework was extended to various issues. NUM was used to model network protocols by “reverse engineering” a given protocol, such as TCP/IP, to provide an “inside look” of the Internet congestion and to obtain fair bandwidth allocation [4][5][6]. In [7][8], delay function influences user’s utility and bandwidth allocation scheme were discussed. Furthermore, a more general utility function that considered NUM and delay in VoIP was discussed in [9] by considering the queuing theory in order to influence the bandwidth allocation by adjusting the delay requirement. This resulted in degrading the performance of lower-priority traffic and the algorithm is VoIP-specific. Delay functions were incorporated into NUM in [11][12][13] by taking the delay function as the network cost which reduced user’s utility. Delay functions in [14][15] were formulated as a ratio between the bandwidth capacity and the buffer occupancy. In this paper, we focus on how to accommodate diverse elastic applications in a network where a mix of traffic may have different requirements for bandwidth and Quality of Service (QoS). As QoS, we consider packet delays and we propose a delay utility function based on \(M/M/1\) queuing results [10].

II. DELAYUTILITY FUNCTION

A. Delay and Utility Function

Let QoS be expressed by the average delay through the network. When a user has the bandwidth \(x\) allocated in a link, it can be considered equivalent to the transmission rate for the user. The \(M/M/1\) based delay function [10], \(d(x)\), is defined as the average delay (including transmission time) in a link:

\[
d(x) = \frac{\alpha}{x_s(x_s - \alpha)} + \frac{1}{x}
\]

\[
= \frac{1}{x_s - \alpha}, \text{ for } x > \alpha \geq 0,
\]

where \(\alpha\) denotes the arrival rate at a link. In this context, \(x_s\) can be interpreted as processing rate of user \(s\). Thus, the delay of the entire path is \(\Sigma r d(x)\), where route \(r\) is a set of link \(l \in L\) that connects source and sink. The delay utility function \(U_{QoS}\) is then formulated so that it increases or decreases according to the delay. This is similar to the idea in [3], where the degree of user’s satisfaction over shorter delay diminishes.

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as the traffic gets smoother. We thus define delay utility function in a single link as follows.

\[ U_{QOS}(x) = m_q \log \left( \frac{1}{d(x)} \right) = m_q \log (x - \alpha), \]  

(2)

where \( m_q \) is the user’s willingness to pay for quality. The delay utility function of a traffic stream traversing a path \( r \) is given by

\[ U_{QOS}(x) = m_q \log \left( \sum_{1 \leq r} x_r - \alpha_r \right), \]  

(3)

The bandwidth allocation problem with the delay QoS is formulated as the following optimization problem.

\[ \text{maximize} \sum_{s \in S} U_s(x_s) \quad \text{s.t.} \quad Hy = x, Ay \leq C, \]  

(4)

where \( U_s(x_s) = U_{bw}^s(x_s) + U_{QOS}^s(x_s) \), user utility function given allocated bandwidth \( x_s \) is

\[ U_{bw}^s(x_s) = m_{w,s} \log(x_s), \]  

(5)

And \( m_{w,s} \) is user \( s \) is willingness to spend on bandwidth \( x_s \) as it is proposed by Kelly in [1][2]. The other variables are identical to those in (2). Furthermore, the allocated bandwidth \( x_s \) must be bounded in the function \( U_{QOS}(x_s) \) because delay \( d(x_s) = \frac{1}{x_s - \alpha_s} > 0 \) must be satisfied. Otherwise, the queue length will grow exponentially, which leads to further performance degradation. Thus, \( U_{QOS}(x_s) \) is modified as follows.

\[ U_{QOS}(x_s) = \begin{cases} m_q \log \left( \sum_{1 \leq r} x_r - \alpha_r \right), & (x_s - \alpha_s) > 0 \\ 0, & \text{Otherwise} \end{cases} \]

4. **Rate Allocation**

The following step, we solve (4) in the Lagrange form, we can write that

\[ L(x, y, z, \lambda, \mu) = \sum_{s \in S} U_s(x_s) - \lambda^T(x - H y) + \mu^T(C - A y - z) \]  

(6)

where \( \lambda = \{ \lambda_s, s \in S \} \) are vectors of Lagrange multipliers and \( \mu = (\mu_j, j \in J) \) is a vector of slack variables. Since (3) and (5) be non-convex as shown in figure 1. Furthermore, notice the derivative of utility function \( U_{QOS}(x_s) \), when \( U_{QOS}(x_s) = 0 \) and \( x_s \leq x_s^{min} \), is \( \frac{dU_{QOS}}{dx_s} = U_{QOS}(x_s) = 0 \). Conversely, \( U'_{QOS}(x_s) > 0 \) when \( x_s > x_s^{min} \). So, when \( x_s > x_s^{min} \),

\[ \int_0^{x_s} U_{QOS}(y) \, dy = \int_0^{x_s} U_{QOS}(y) \, dy + \int_{x_s}^{x_s^{min}} U_{QOS}(y) \, dy = \int_{x_s}^{x_s^{min}} U_{QOS}(y) \, dy > 0. \]

As we observed, when \( x_s > x_s^{min} \), utility function \( U_{QOS}(x_s) \) is an increasing and strictly concave function, which is convex. So, that allows the addition of two convex functions, \( U_{bw}^s(x_s) \) and \( U_{QOS}^s(x_s) \), is also convex. Therefore, for \( U_{QOS}^s(x_s) \) to be convex, it must satisfy \( x_s > x_s^{min} \).

For that reason, to preserve convexity and prevent from becoming into non-convex problem, the problem of utility maximization is reformulated as follows.

\[ \text{Maximize} \sum_{s \in S} U_s(x_s) \quad \text{s.t.} \quad Ax \leq C, \]  

\[ x - x_s^{min} \geq 0, \]  

\[ \text{over} \ (x^{min} \geq 0). \]

Therefore, in this thesis, we assume the utility functions are continuous and twice differentiable on \((0, +\infty)\) and satisfy the following properties:

1. \( U_{bw}(x_s), U_{QOS}(x_s) \geq 0, \forall x_s. \)
2. \( x_s, 0 \leq x_s^{min} \leq x_s \) and \( U_{bw}(0), U_{QOS}(0) = 0, \forall x_s, s. \)
3. \( U_{bw}(x_s) \) and \( U_{QOS}(x_s) \) are twice differentiable.
4. \( U_{bw}(x_s) \) and \( U_{QOS}(x_s) \) are concave.
5. \( \frac{dU_{bw}(x_s)}{dx_s}, \frac{dU_{QOS}(x_s)}{dx_s} < 0, \forall s \leq 0 \leq x_s \leq c. \)
6. \( \lim_{x_s \to 0} \frac{dU_{bw}(x_s)}{dx_s}, \lim_{x_s \to 0} \frac{dU_{QOS}(x_s)}{dx_s} < 0, \forall s. \)
7. \( \sum_{s \in S} x_s^{min} \leq C_i. \)
8. \( \forall x_s, \sum_{s \in S} x_s^{min} \leq \sum_{s \in S} x_s \leq C_i, \forall x_s^{min} < x_s. \)

The property 7 and 8 assure that \( U_{QOS}(x_s) \) convexity and network satisfied of the minimum bandwidth requirement.

4. **Rate Allocation**

The following step, we solve (4) in the Lagrange form, we can write that

\[ L(x, y, z, \lambda, \mu) = \sum_{s \in S} U_s(x_s) - \lambda^T(x - H y) + \mu^T(C - A y - z) \]  

(6)
are convex functions, the Lagrangian form can be directly solved such that
\[
\frac{dU_s}{dx_s} = \frac{dU_{bw}^s}{dx_s} + \frac{dU_{qos}^s}{dx_s} = \frac{m_{x,s}}{x_s} + \frac{m_{q,s}}{x_s - a_s}. \tag{7}
\]
By combining the derivative of (6) and (7), we have
\[
\frac{dL}{dx_s} = \frac{m_{x,s}}{x_s} + \frac{m_{q,s}}{x_s - a_s} - \lambda_s = 0, \tag{8}
\]
which yields \( \lambda_s = \frac{x_s m_{x,s} - a_s m_{x,s}^x + x_s m_{q,s}}{x_s (x_s - a_s)} \), and
\[
x_s = \frac{(x_s m_{x,s} - a_s m_{x,s}^x + x_s m_{q,s}) \pm \sqrt{(x_s m_{x,s} - a_s m_{x,s}^x + x_s m_{q,s})^2 - 4 x_s a_s m_{x,s}}}{2 x_s}. \tag{9}
\]
The solution of \( x_s \) can be obtained from
\[
\sum x_s = c_l, \tag{10}
\]
By combining (9) to (10) and solving for \( \lambda_s \) on link \( l \), for all user sharing link \( l \), \( l \in \mathcal{L} \). It can be interpreted as deciding the network price that users must pay for using link \( l \). Once \( \lambda_s \) is obtained, user \( s \) solves \( x_s \) by manipulating of (8),
\[
x_s = \frac{x_s m_{x,s} - a_s m_{x,s}^x + x_s m_{q,s}}{\lambda_s} + a_s.
\]
Additionally, network may also decide the minimum value for \( m_t \) and \( m_q \) to offset the operation cost. For instance,
\[
m_{x,s} = \max \left( m_{x,s}, \sum_{l \in \mathcal{L}} \frac{\text{cost}(l)}{n_l} \right) \tag{11}
\]
and
\[
m_{q,s} = \max (m_{q,s}, \text{cost}(q)), \tag{12}
\]
where \( \text{cost}(l) \) denotes the operation cost on link \( l \) [16]. \( n_l \) is the number of users sharing \( l \), \( \text{cost}(q) \) is the operation cost to acquire quality \( q \), and for \( m_{x,s}, m_{q,s}, \text{cost}(l), \text{cost}(q) \geq 0 \). The design of cost function \( \text{cost}(.) \) is beyond the scope of our discussion. Additionally, user may be paying too much when
\[
\frac{U_s(x_s)}{m_{x,s} + m_{q,s}} < \text{Threshold}.
\]
In [1], Kelly has also introduced a concept of fairness.

**Definition 1:** A vector of rates \( x = (x_l, l \in \mathcal{L}) \) is proportionally fair if it is feasible, that is \( x \geq 0 \) and \( Ax \leq C \), and if for any other feasible vector \( x^* \), the aggregate of proportional changes is zero or negative:
\[
\sum_{s \in S} \frac{(x^*_s - x_s)}{x_s} \leq 0. \tag{13}
\]
However, (13) is not sufficient when \( x^* \leq \lambda \) because delay function \( d(x^*) = \frac{1}{x^* - a} \leq 0 \). Thus,
\[
x^* = \begin{cases} x^*, & \text{if } x^* - a > 0 \\ x, & \text{Otherwise} \end{cases} \tag{14}
\]

**Corollary 1:** Condition (14) satisfies (13).

**Proof:** There is other feasible vector \( x^* \) that is proportionally fair and \( x^*_s \leq a_s \). But by condition in (14), \( x^*_s = x_s \) then \( \sum_{s \in S} \frac{(x^*_s - x_s)}{x_s} = \sum_{s \in S} \frac{(x^*_s - x_s)}{x_s} = 0 \). Otherwise \( \sum_{s \in S} \frac{(x^*_s - x_s)}{x_s} \leq 0 \). Thus satisfies (13). \( \blacksquare \)

### III. Result and Analysis

#### A. The Impact of User Willingness to Pay

Consider a simple configuration with two nodes connected by a single link \( l \) with capacity \( C=100 \). The link \( l \) is shared by flows \( a \) and \( b \). In order to investigate the effect of \( \lambda \), we assume that \( m_{a,l} = m_{b,l} = 10 \) and \( m_{q,a} = m_{q,b} = 10 \), initially. When \( a_a = 60 \) and \( a_b = 1 \), the initial bandwidth allocation equally divides the excess capacity between the two flows \( \frac{c - a_a - a_b}{2} = 19.5 \) such that \( x_a = 79.5 \) and \( x_b = 20.5 \).

**Fig. 2:** the relationship of bandwidth distribution between user \( a \) and \( b \).

In Fig. 2, we plot bandwidths as \( m_{q,b} \) (the willingness to pay for QoS) increases for flow \( b \). The figure shows that bandwidth for \( b \) increases (for \( a \) decreases) and converges.

#### B. Parking Lot Configuration

As a potentially congested example, a parking lot configuration with four nodes is considered.

**Fig. 3:** Single Bottleneck
Three flows $a$, $b$, and $c$ share links as depicted in the fig. 3. they have the same values for $m_x$ and $m_q$, as in Table 1, the bandwidth is identical ($x_a = x_b = x_c = 33.333$) as they equally share the link $C_{CD}$. We assume that link capacities are identical in three links with $C_{AB} = C_{BC} = C_{CD} = 100$. For simplicity, $\lambda_x = \max\{\lambda_{s(1)}\} \ l \in r_2$.

<table>
<thead>
<tr>
<th>USER $a$, $b$, $c$</th>
<th>$M_x$</th>
<th>$M_0$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>flow 1</td>
<td>10</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1 (case 1)

Suppose that flow $c$ increases the willingness to pay from $m_{q,c} = 5$ to 50 and 100 (it remains the same in other flows). The resulting bandwidths in three flows are listed in Table 2.

The configuration of the network is illustrated in Fig. 4, where $m_x$, $m_q$, and $\alpha$, and the result from bandwidth allocation $x$ for each flow and its delay is listed in Table 3. The delay of each flow $d(x)$ is the summation of delay occurrence on each link along the path which the flow traverses.

<table>
<thead>
<tr>
<th>flow 1</th>
<th>flow 2</th>
<th>flow 3</th>
<th>flow 4</th>
<th>flow 5</th>
<th>flow 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_x$</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>$m_q$</td>
<td>10</td>
<td>10</td>
<td>20</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>20</td>
<td>20</td>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>$x$</td>
<td>23.51</td>
<td>27.51</td>
<td>17.45</td>
<td>27.31</td>
<td>30.72</td>
</tr>
<tr>
<td>$d(x)$</td>
<td>1.12</td>
<td>0.39</td>
<td>0.26</td>
<td>0.23</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 3

This example illustrates the usage of willingness to pay and user’s demand for bandwidth to influence the flow’s delay. For example, flow 4 and 6 are example of flows with high QoS demand, where flow 1 and 2 are flows with lower QoS demand. Additionally, flow can also only require bandwidth without considering the quality like flow 5. Moreover, flow 5 is a special case of unusually high demand for bandwidth allocation without QoS guarantee. Notice that flow 3 can reduce its delay if flow 3 traverses over link $AD$ instead of $AC$ and $CD$ because less bottleneck in $AD$ and less number of hops from source to sink. For instance, flow 3’s delay can be reduced from $d(x_3) = 0.26$ to 0.039 and achieves higher utility if flow 3 traverses over $AD$.

### IV. Conclusion

In this paper, we address the impact of incorporating QoS to utility function and present a pricing scheme which takes user’s requirement for QoS into consideration. On the other hand, our model does not consider propagation delay because it is assumed to be constant. In addition, queuing model provides long term average value and input data is measured over an extended period of time. Furthermore, the proposal model only supports elastic traffic like email, FTP, HTTPs, and others like these. However, we need to consider real time traffic, which is a non-convex, and the extension of the model to include non-convex traffic will be the topic of further studies.

### References


