Exam Time: 1h & 15m. Each problem is worth 10 points. 50 pts total. Choose any 5 problems.

1. **Regular Expressions and Nondeterministic Finite Automata.**
   
a) Give a regular expression for the set of all strings over $\Sigma = \{0,1\}$ having an odd number of zeros.

   b) Construct an NFA that recognizing the language defined by this regular expression. Use the labeled digraph method.

   **Hints**: The regular expression is $1^*01^*(01^*01^*)^*$. For the construction: start with two vertices, say $s$ and $f$ and an arrow connecting them, labeled by the RE:
Let \( \{p_0, p_1, p_2, p_3\} \) be the states of the first automaton, \( \{q_0, q_1, q_2, q_3\} \) those of the second. The product automaton will have states \( \{(p_i, q_j) \mid 0 \leq i \leq 3, 0 \leq j \leq 3\} \), \( F = F_p \times F_q = \{[p_3, q_0], [p_3, q_1], [p_3, q_2]\} \). The transition function can be read off the two automata, the start state is \([p_0, q_0]\).

3. **NFA to DFA.** Given the NFA:

![NFA Diagram]

convert it into an equivalent DFA, showing all intermediate steps.

**Hints**: Textbook, Section 2.4. REMEMBER you must always include the results of the \( \epsilon \)-transitions.

4. **Myhill-Nerode Theorem.**
   a) Define the relation \( R_L \) used by the theorem.
   b) Find all equivalence classes of \( R_L \) for the language \( \{x \in \{0,1\}^* \mid \#_0(x) \neq \#_1(x)\} \), where \( \#_a(w) \) is the number of occurrences of the symbol \( a \) in \( w \).
   c) What does the Myhill-Nerode theorem tell you about this language and why? Hint: you may need to quote the theorem...
Hints: a) Text, p. 70.: for any language \( L \subseteq \Sigma^* \), a relation \( R_L \) on \( \Sigma^* \):

\[
x R_L y \iff (\forall w)[xw \in L \iff yw \in L]
\]

b) Equivalence classes: \([x]_{R_L} = \{ \text{all those strings such that the difference between the number of zeros and of ones is the same as that of } x \} \). For any two strings \( x, y \) for which this difference is NOT the same, there exists a suffix string such \( w \) that \( xw \in L \) and \( yw \) is not in \( L \). Since the difference can be \( 0, \pm 1, \pm 2, \ldots \), the number of such classes is infinite.

c) Since the Myhill-Nerode Theorem asserts that a language \( L \) is regular if and only if \( R_L \) partitions \( \Sigma^* \) into a finite number of equivalence classes, the language in question is not regular.

5. Language Properties. Define the complementary or \( \text{cor} \) of two languages by

\[
\text{cor}(L_1, L_2) = \{ w : w \in \bar{L}_1 \text{ or } w \in \bar{L}_2 \}.
\]

Prove that the family of regular languages is closed under the \( \text{cor} \) operation.

Hints: \( \bar{L}_1 \cup \bar{L}_2 = \bar{L}_1 \cap L_2 \). The result follow immediately from the prior results: the complement of a regular language is regular; the intersection (union) of two regular languages is regular.

6. Minimum DFAs. You are given the NFA

\[
\begin{array}{c}
\text{a} \\
\text{b} \\
\text{c}
\end{array}
\]

Construct a minimum DFA equivalent to it. Explain the construction.

Hints: this corresponds to the language \( \{ a^* b^* c^* \} \).

\( R_L \) has the following classes: \([a]_{R_L} \) which consists of all the strings of the form \( a^* \); \([b]_{R_L} \) which consists of all strings of the form \( a^* b^+ \); \([c]_{R_L} \) which consists of all strings of the form \( a^* b^+ c^+ \). The remaining class is that of all those strings that do not belong to one of the three classes already identified. We need to show that any strings \( x \) and \( y \) not belonging to one of the three classes satisfy \( x R_L y \). We observe that any such string \( x \) must have either an \( a \) following a \( b \), or an \( a \) following a \( c \) or a \( b \) following a \( c \). In any such case there is no suffix that will produce a string of \( L \), so \( xw \in L \iff yw \in L \) since neither can ever be in \( L \).

\( \text{Index}(R_L) = 4 \), and the minimum DFA has 4 states. Any four state DFA that accepts the language is minimal: for example