4.2)

Let $E_{\text{DFA,REX}} = \{<A,R> | A \text{ is a DFA, } R \text{ is a regular expression and } L(A) = L(R)\}$. The following TM $E$ decides $E_{\text{DFA,REX}}$.

$E = \text{"On input } <A,R>:\text{\ "}
  1. Convert regular expression $R$ to an equivalent DFA $B$.
      • Use the method of Lemma 1.55 to convert $R$ to an NFA $N$
      • Use the method of Theorem 1.39 to convert $N$ to a DFA $B$
  2. Use TM $C$ of Theorem 4.5 for deciding $E_{\text{DFA}}$ on input $<A,B>$.
  3. If $C$ accepts, accept. If $C$ rejects, reject.”

Explanation:
The trick here is simple—convert $R$ to a DFA and use the decider for $E_{\text{DFA}}$ to decide $E_{\text{DFA,REX}}$.

4.3)

Let $\text{ALL}_{\text{DFA}} = \{<A> | A \text{ is a DFA that recognizes } \sum^* \}$. The following TM $L$ decides $\text{ALL}_{\text{DFA}}$.

$L = \text{"On input } <A> \text{ where } A \text{ is a DFA:}\text{\ "}
  1. Construct DFA $B$ that recognizes the complement of $L(A)$ as described in Exercise 1.14a.
  4. Run TM $T$ of Theorem 4.4 on input $<B>$, where $T$ decides $E_{\text{DFA}}$.
  5. If $T$ accepts, accept. If $T$ rejects, reject.”

Explanation:
The complement of $\sum^*$ is $\emptyset$, the empty language. Not to be confused with $\varepsilon$, the empty string, which is in $L(A) = \sum^*$.

4.4)

Let $A_{\varepsilon_{\text{CFG}}} = \{<G> | G \text{ is a CFG that generates } \varepsilon \}$. The following TM $V$ decides $A_{\varepsilon_{\text{CFG}}}$.

$V = \text{"On input } <G> \text{ where } G \text{ is a CFG:} \text{\ "}$
1. Run TM S from Theorem 4.7 on input $<G, \varepsilon>$, where S is a decider for $A_{CFG}$.
   2. If S accepts, accept. If S rejects, reject.”

Another equally good solution was provided by some students:
1. Convert G to Chomsky Normal Form.
2. If G has the rule $S \rightarrow \varepsilon$, then accept, else reject.