Name: ______________________________________________________

Points: 1.______ 2.______ 3.______ 4.______ 5.______ 6.______ 7.______ 8.______

Total ______/60.

Each problem is worth 10 points - 60 pts total.

Do 1; do four of 2-7; do 8. If you attempt more than six, dentify clearly which five problems you wish graded.

1. **Languages.** prove or disprove the claim

\[(L_1 \cup L_2)^R = L_1^R \cup L_2^R\]

for all languages \(L_1\) and \(L_2\). Recall that \(R\) denotes reversal.

**Solution.** We need to prove two distinct things

1. If \(x \in (L_1 \cup L_2)^R\) then \(x \in L_1^R \cup L_2^R\).
2. If \(x \in L_1^R \cup L_2^R\) then \(x \in (L_1 \cup L_2)^R\).

We shall prove the first, leaving the second as an exercise. If \(x \in (L_1 \cup L_2)^R\) then \(x^R \in ((L_1 \cup L_2)^R)^R = L_1 \cup L_2\). This implies that \(x^R \in L_1\) or \(x^R \in L_2\), and this, in turn, implies that \(x \in L_1^R\) or \(x \in L_2^R\). The first inclusion follows. The second one should now be easy.

2. **Deterministic Finite Automata.** (7 pts) For \(\Sigma = \{a, b\}\), construct a DFA that accepts all strings with no more than 3 a’s. (3 pts) Construct a DFA that will accept the complementary language.
3. **Non-Deterministic Finite Automata.** Design an NFA with no more than five states for the set

\[ \{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}.\]
4. **NFA to DFA.** Given the NFA:

![NFA Diagram]

convert it into an equivalent DFA. Show all the steps of the construction.

**Solution:** Note: e stands for $\epsilon$. Note further that the $\epsilon$-closure of $\{q_0\}$ is $\{q_0, q_1\}$. The resulting states are always computed by an $\epsilon$-closure. Start with $q_0 \to \{q_0\} \to \{q_0, q_1\} = q_{01}$ and continue from there, building the table:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>${q_0, q_1}$</td>
<td>${q_0, q_1, q_2}$</td>
<td>${q_1}$</td>
</tr>
<tr>
<td>${q_0, q_1, q_2}$</td>
<td>${q_0, q_1, q_2}$</td>
<td>${q_1}$</td>
</tr>
<tr>
<td>${q_1}$</td>
<td>${q_0, q_1, q_2}$</td>
<td>${q_1}$</td>
</tr>
</tbody>
</table>

![DFA Diagram]

DFA.

Can you construct a simpler DFA?

5. **Regular Expressions.** For $\Sigma = \{a, b, c\}$, give a regular expression for the language over $\Sigma$ whose strings contain no more than three $a$’s.
Solution:

\[(b \cup c)^* (\epsilon \cup a)(b \cup c)^* (\epsilon \cup a)(b \cup c)^* (\epsilon \cup a)(b \cup c)^*\].

6. Regular Expressions.

(5 pts) Construct an NFA that accepts the language:

\[L = L(ab^*a^*) \cup L((ab)^*ba)\]

(5 pts) From this NFA construct the appropriate GNFA and derive another equivalent regular expression.

Solution: we construct the NFAs for the two languages, then combine them into a single NFA. We then construct the GNFA. We conclude with the reduction to a regular expression. Note that \(e\) replaces \(\epsilon\) and \(+\) replaces \(\cup\).

1. The NFA for \(L(ab^*a^*)\):

2. The NFA for \(L((ab)^*ba)\).

3. The GNFA
You can then reduce the GNFA one node at a time... Reduce $q_0$.

Reduce $q_0$. 
Etc.

7. **Language Properties.** Sketch the **constructive** proof of the theorem:
   If \( L_1 \) and \( L_2 \) are regular languages, then \( L_1 \cap L_2 \) is a regular language.

   **Solution:** same proof method as for union; careful about taking only accepting pairs of states from the original DFAs to make up the accepting states of the *intersection acceptor*.

8. **Pumping Lemma.**

   (5 pts) State the Pumping Lemma.

   (5 pts) Using the Pumping Lemma, prove that the language \( \{0^n1^n0^n \mid n \geq 0\} \) is not regular. Make sure you explain the process.

   **Solution:** Slide 20, Ch. 3, Sect. 4. Ex. 1.73 of text.