<table>
<thead>
<tr>
<th>For all motion:</th>
<th>For uniform acceleration only:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_i - x_0}{t_i - t_0} )</td>
<td>3. ( x(t) = \frac{1}{2}a_xt^2 + v_{xo}t + x_o )</td>
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<tr>
<td>2. ( \bar{a} = \frac{\Delta \bar{v}}{\Delta t} = \frac{v_i - v_0}{t_i - t_0} )</td>
<td>6. ( \Delta x = \frac{1}{2} \left( v_1 + v_0 \right) \Delta t ) or ( \Delta x = \frac{v_1 + v_0}{2} )</td>
</tr>
<tr>
<td>[ 4. \ v_x(t) = a_xt + v_{xo} ]</td>
<td>[ 7. \ v_i^2 = v_0^2 + 2a\Delta x ]</td>
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<tr>
<td>[ 5. \ a_x(t) = a_x ]</td>
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**Bonus: (+5 points)** If you solve equation #5 for \( t \), then substitute for \( t \) into equation #4 and simplify, what is the result? You should get a familiar result!

60. A parachutist descending at a speed of 10 m/s loses a shoe at an altitude of 50.0 m.
   a. **When** does the shoe reach the ground?
   b. **What** is the **velocity** of the shoe just before it hits the ground?

**Solution:** The book expects you to assume freefall conditions: the only force acting on the shoe is gravity (no air resistance, a.k.a. drag), so \( a_y = -g = -9.81 \text{ m/s}^2 \). But realistically, if it were freefall, the parachutist would not have any wind resistance to slow her down!

\[
\begin{align*}
\text{Diagram @ t = 0s:} & \\
\text{Initial conditions:} & \\
\text{t}_0 = 0 \text{ s} & \\
y_o = +50 \text{ m} & \\
v_{yo} = -10 \text{ m/s} & \\
a_y = -9.81 \text{ m/s}^2 & \\
\text{Motion Eqns:} & \\
y(t) = -4.905t^2 - 10t + 50 & [\text{m}] \\
v_y(t) = -9.81t - 10 & [\text{m/s}] \\
a_y(t) = -9.81 & [\text{m/s}^2]
\end{align*}
\]

(a) When the shoe reaches the ground, \( y = 0 \), so set \( y(t) = 0 \) and solve for \( t \) using the quadratic eqn...

\[
0 = -4.905t^2 - 10t + 50 \rightarrow t = +2.33 \text{ s} \text{ and } -4.37 \text{ s}
\]

Reject the negative answer; if you graph \( y(t) \) you'll understand why – ask if you don't!

(b) Substitute answer from (a) into \( v(t) \) eqn. and evaluate...

\[
v(t = +2.33) = -9.81(+2.33) - 10 = -32.88 \text{ m/s}
\]
61. A mountain climber stands at the top of a 50.0 m cliff hanging over a calm pool of water. The climber throws two stones vertically 1.0 s apart and observes the cause a single splash when they hit the water. The first stone has an initial velocity of + 2.0 m/s.
   a. How long after the release of the first stone will the two stones hit the water?
   b. What is the initial velocity of the second stone when it is released?
   c. What will the velocity of each stone be at the instant both stones hit the water?

**Solution:** Assume freefall conditions \((g = 9.81 \text{ m/s}^2 \text{ downward, not other forces})\)

**Diagram @ t = 0s:**

\[v_{yo} = +2 \text{ m/s}\]

\[y_o = +50 \text{ m}\]

\[0 \text{ m}\]

**Initial conditions:**

<table>
<thead>
<tr>
<th></th>
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<th>②</th>
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</thead>
<tbody>
<tr>
<td>(t_0)</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(y_0)</td>
<td>+50</td>
<td>+50</td>
</tr>
<tr>
<td>(v_{yo})</td>
<td>+2</td>
<td>??</td>
</tr>
<tr>
<td>(a_y)</td>
<td>−9.81</td>
<td>−9.81</td>
</tr>
</tbody>
</table>

**Motion Eqns:**

**Stone #1**

\[y(t) = -4.905t^2 +2t + 50 \quad [\text{m}]\]

\[v_y(t) = -9.81t + 2 \quad [\text{m/s}]\]

\[a_y(t) = -9.81 \quad [\text{m/s}^2]\]

**Stone #2:** (adjusted for 1s delay)

\[y(t–1) = -4.905(t–1)^2 + v_{yo}(t–1)+ 50 \quad [\text{m}]\]

\[v_y(t) = -9.81(t–1) + v_{yo} \quad [\text{m/s}]\]

\[a_y(t) = -9.81 \quad [\text{m/s}^2]\]

Both stones are synched to the same stopwatch that starts at \(t = 0s\). Stone 2’s equations are shifted to account for 1s delay.

(a) When the stone reaches the pond, \(y = 0\), so set \(y(t) = 0\) and solve for \(t\) using the quadratic eqn...

\[0 = -4.905t^2 +2t + 50 \rightarrow t = +3.40s \text{ and } -3.00s\]

*Reject the negative answer; if you graph \(y(t)\) you’ll understand why – ask if you don’t!*

(b) When the 2\(^{nd}\) stone hit the pond, you know its height (0m) and the time it hits (3.40s). When you plug (0m, 3.40s) into stone #2’s position equation, you know everything but \(v_{yo}\), so plug & chug!

\[y(t–1) = -4.905(t–1)^2 + v_{yo}(t–1)+ 50\]

\[0 = -4.905(3.40s–1)^2 + v_{yo}(3.40s –1)+ 50\]

\[v_{yo} = -9.03 \text{ m/s}\]

You can check this answer by graphing both position eqns to see if the stones land simultaneously!

(c) Substitute \(t = +3.40s\) from part (s) and the answer from (b) into \(v(t)\) eqns. for both stones and evaluate...

**Stone 1:**

\[v_y(t) = -9.81t + 2\]

\[v_y(t) = -9.81(3.40s ) + 2\]

\[v_y(t) = -33.4 \text{ m/s}\]

**Stone 1:**

\[v_y(t) = -9.81(t–1) – 9.03 \text{ m/s}\]

\[v_y(t) = -9.81(3.40s –1) – 9.03 \text{ m/s}\]

\[v_y(t) = – 32.57 \text{ m/s}\]
62. A model rocket is launched straight upward with an initial speed of 50.0 m/s. It accelerates with a constant upward acceleration of +2.0 m/s² until its engines stop at an altitude of 150 m.

a. What is the maximum height reached by the rocket? (a.k.a. “apogee”)
b. When does the rocket reach its maximum height?
c. How long is the rocket in the air?

**Solution:** You need to break this problem down into two parts (a piecewise function):

1. When the engine is running from \( y = 0 \) to \( 150 \) m; note the +2 m/s² acceleration is net of gravitational acceleration.
2. When the engines have stopped (i.e., rocket begins freefalling) – what time does this happen? What is \( v_{yo} \)?

**Diagram @ t = 0s:**

![Diagram](image)

- \( v_{yo} = ?? \text{ m/s} \)
- \( y_o = +150 \text{ m} \)
- \( y_o = 0 \text{ m} \)

**Initial conditions**

- \( t_0 = 0 \) \( \text{[s]} \)
- \( y_0 = 0 \) \( +150 \) \( \text{[m]} \)
- \( v_{yo} = +50 \) \( v_{yo} ?? \) \( \text{[m/s]} \)
- \( a_y = +2 \) \( -9.81 \) \( \text{[m/s}^2] \)

**Motion Eqns:**

- **Stage 1**
  \( y(t) = +1t^2 +50t + 0 \) \( \text{[m]} \)
  \( v_y(t) = +2t + 50 \) \( \text{[m/s]} \)
  \( a_y(t) = +2 \) \( -9.81 \) \( \text{[m/s}^2] \)

- **Stage 2**
  \( y(t) = –4.905t^2 + v_{yo}t + 150 \) \( \text{[m]} \)
  \( v_y(t) = –9.81t + v_{yo} \) \( \text{[m/s]} \)
  \( a_y(t) = -9.81 \) \( \text{[m/s}^2] \)

**Piece 1:** first figure out what you can determine for the “engine on” stage from \( y_o = 0 \text{m} \) to \( y_1 = 150 \text{m} \)...

If you look at the motion equations and check off your “knowns” you’ll find two useful equations...

**Equation #3 can be solved for time to reach 150m:** You know everything but \( t \), so you can plug in your values and solve for \( t \) using quadratic equation (you could also graph and look for time it reaches 150m). I’m going to demonstrate what I mean by finding a “general solution” that could be used over and over again for future problems:

\[
y_1 = \frac{1}{2}a_yt^2 + v_{yo}t + y_o
\]

\( 0 = \frac{1}{2}a_yt^2 + v_{yo}t + y_o - y_1 \) (subtract \( y_1 \) from both sides)

\( 0 = \frac{1}{2}a_yt^2 + v_{yo}t - \Delta y \) (Note that \( y_o - y_1 = -y_1 + y_o = -(y_1 - y_o) = -\Delta y \))

\[
t = \frac{-v_{yo} \pm \sqrt{v_{yo}^2 - 4 \cdot \frac{1}{2}a_y \cdot -\Delta y}}{2 \cdot \frac{1}{2}a_y} = \frac{-v_{yo} \pm \sqrt{v_{yo}^2 + 2a_y\Delta y}}{a_y} = \frac{-50 \text{m/s} \pm \sqrt{(50 \text{m/s})^2 + 2(+2 \text{m/s}^2)(+150 \text{m})}}{+2 \text{m/s}^2}
\]

You should note that the units in the equation above are consistent – an important check for accuracy!

\( t = +2.84 \text{s} \) and \( -52.83 \text{s} \) (reject the negative time – why??)

**Equation #7 can be solved for \( v_1 \):** Once you know \( v_1 \), you will have enough info to complete your motion equations for stage 2 of the rocket:

\[
v_1^2 = v_0^2 + 2a\Delta x \quad \text{you know everything but } v_1 \text{ – isolate } v_1 \text{ then plug & chug!}
\]

\[
v_1 = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{(50 \text{m/s})^2 + 2(+2 \text{m/s}^2)(+150 \text{m})} = +55.7 \text{ m/s}
\]
Problem #62, continued...
So here are the revised motion equations for Stage 2:

\[ y(t) = -4.905t^2 + 55.7t + 150 \text{ [m]} \]

The rocket is now in freefall. You want to find the max height, and you know that when the rocket reaches its max height, its instantaneous velocity is zero! You can do a couple of things with this info...

You could set \( v = 0 \) in the second motion equation above, then solve for the time:

\[ v_f(t) = -9.81t + 55.7 \]

\[ 0 = -9.81t + 55.7 \]

\[ t = \frac{55.7}{9.81} = 5.68 \text{s} \]

So time to max height time = 2.84s + 5.68s = 8.52s

Now use first equation above to find height @ t = 5.68:

\[ y(t = 5.68s) = -4.905(5.68s)^2 + 55.7(5.68s) + 150 \]

\[ y(t = 5.68s) = 308.1 \text{ m} = \text{answer to part (a)} \]

To answer the last part of the question (how long is the rocket in the air), use the \( y(t) \) equation above to find when the rocket hits the ground. Set \( y = 0 \) and use the quadratic equation to solve for \( t \):

\[ 0 = -4.905t^2 + 55.7t + 150 \]

\[ t = 13.60 \text{ and } -2.24 \text{ (reject negative)} \]

So total time in air = 2.84s + 13.60s = 16.44s (answer to part c).

63. A professional race car driver buys a car ① that can accelerate at \(+5.9 \text{ m/s}^2\). The racer decides to race against another driver in a souped-up stock car ②. Both start from rest, but the stock-car driver leaves 1.0 s before the driver of the sports car. The stock car ② moves at a constant acceleration of \(+3.6 \text{ m/s}^2\).

a. Find the time it takes the sports-car driver ① to overtake the stock car driver ②.

b. Find the distance the two drivers travel before they are side by side.

c. Find the velocities of both cars at the instant they are both side by side.

We solved this in class – check your notes.
Two cars are traveling along a straight line in the same direction, the lead car ① at 25 m/s and the other car ② at 35 m/s. At the moment the cars are 45 m apart, the lead driver applies the brakes, causing the car to have an acceleration of \(-2.0 \text{ m/s}^2\).

a. How long does it take for the first car ① to stop?

b. Assume that the driver of the chasing car ② applies the brakes at the same time as the driver of the lead car. What must the chasing car’s minimum negative acceleration be to avoid hitting the lead car ①?

c. How long does it take for the chasing car ② to stop?

**Diagram @ t = 0s:**

\[
\begin{array}{c}
\text{Car } ①: \quad V_{x0} = +25 \text{ m/s} \\
\text{Car } ②: \quad V_{x0} = +35 \text{ m/s}
\end{array}
\]

\[
\begin{array}{c}
a_x = -2 \text{ m/s}^2
\end{array}
\]

**Solution:**

(a) The velocity of car #1 will be \(v_1 = 0\) when the car stops. Using this info, you could use \(v_f^2 = v_i^2 + 2a\Delta x\) to find the distance to stop (\(\Delta x\)), then use that result in \(\Delta t = \frac{1}{2}(v_i + v_f)\Delta t\) to find the time it takes to stop. Try it!

Alternatively, you could use the \(v(t)\) eqn for car 1 and fact that \(v_1 = 0\) to find the stopping time, and you could use the \(x(t)\) equation to find stopping distance...

\[v_f(t) = -2t + 25\]
\[0 = -2t + 25\]
\[t = 25/2 = 12.5\text{s}\]

\[x = 201.25\text{ m}\]

(b) If the driver of car 2 hit the brakes at the same time, he has 201.25m to come to a complete halt. Use motion equation #7:

\[v_f^2 = v_i^2 + 2a\Delta x\]

\[a = \frac{v_f^2 - v_i^2}{2\Delta x} = \frac{0^2 - (35 \text{ m/s})^2}{2(201.25\text{ m})} = -3.04 \text{ m/s}^2\]

Note that the units on the calculation are consistent and the sign is negative as expected. This rate of deceleration will stop the cars bumper-to-bumper. Anything more will put some space between the cars.

(c) To answer part (c) you could use your position equation for car 2...

\[201.25\text{ m} = -\frac{1}{2}(-3.04\text{ m/s}^2)^2 + 35t + 0\]

and then use the quadratic eqn to solve for \(t\).

OR you could use motion equation #6:

\[\Delta x = \frac{1}{2}(v_i + v_f)\Delta t\]

\[\Delta t = \frac{2\Delta x}{(v_i + v_f)} = \frac{2(201.25\text{ m})}{(0 + 35 \text{ m/s})} = 11.5 \text{ s}\]
65. One swimmer in a relay race has a 0.50 s lead and is swimming at a constant speed of 4 m/s. The swimmer has 50.0 m to swim before reaching the end of the pool. A second swimmer moves in the same direction as the leader. What constant speed must the second swimmer have in order to catch up to the leader at the end of the pool?

Diagram @ t = 0s:

\[ \begin{align*}
\text{Initial conditions} & \quad \text{Car } 1 & \quad \text{Car } 2 \\
& & \\
05 \text{ s behind } & -50 \text{ m} & 0 \text{ m} \\
& & +x \\
& & \\
\end{align*} \]

\[ \begin{align*}
& t_0 = 0 \text{ [s]} \\
& x_0 = -50 \text{ [m]} \\
& v_{x0} = +4 \text{ [m/s]} \\
& a_x = 0 \text{ [m/s}^2\text{]} \\
\end{align*} \]

\[ \begin{align*}
& \text{Motion Eqns: Car } 1 \\
x(t) = 0t^2 + 4t - 50 \text{ [m]} \\
v_x(t) = 0t + 4 \text{ [m/s]} \\
a_x(t) = 0 \text{ [m/s}^2\text{]} \\
\end{align*} \]

\[ \begin{align*}
& \text{Car } 2 \\
x(t) = 0t^2 + v_{yo}t + y_0 \text{ [m]} \\
v_x(t) = 0t + v_{yo} \text{ [m/s]} \\
a_x(t) = 0 \text{ [m/s}^2\text{]} \\
\end{align*} \]

I set up the problem...you finish it!