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Harris Corner Detector
Finding Corners

Edge detectors perform poorly at corners.

Corners provide repeatable points for matching, so are worth detecting.

Idea:

- Exactly at a corner, gradient is ill defined.
- However, in the region around a corner, gradient has two or more different values.

The Harris corner detector

Form the second-moment matrix:

\[
C = \begin{bmatrix}
\sum I_x^2 & \sum I_x I_y \\
\sum I_x I_y & \sum I_y^2
\end{bmatrix}
\]

Sum over a small region around the hypothetical corner

Gradient with respect to x, times gradient with respect to y

Matrix is symmetric
Simple Case

First, consider case where:

\[ C = \begin{bmatrix}
\sum I_x^2 & \sum I_x I_y \\
\sum I_x I_y & \sum I_y^2
\end{bmatrix} = \begin{bmatrix}
\lambda_1 & 0 \\
0 & \lambda_2
\end{bmatrix} \]

This means dominant gradient directions align with x or y axis.

If either \( \lambda \) is close to 0, then this is not a corner, so look for locations where both are large.

General Case

It can be shown that since \( C \) is symmetric:

\[ C = R^{-1} \begin{bmatrix}
\lambda_1 & 0 \\
0 & \lambda_2
\end{bmatrix} R \]

So every case is like a rotated version of the one on last slide.
So, To Detect Corners

- Filter image
  - with Gaussian to reduce noise
- Compute magnitude of the gradient everywhere
  - magnitude of the x and y gradients at each pixel
- Construct C in a window
  - Construct C in a window around each pixel (Harris uses a Gaussian window – just blur)
- Use Linear Algebra to find $\lambda_1$ and $\lambda_2$
  - Solve for product of $\lambda$s (determinant of C)
- If they are both big, we have a corner
  - If $\lambda$s are both big (product reaches local maximum and is above threshold), we have a corner (Harris also checks that ratio of $\lambda$s is not too high)
Corner Detection

Corners are detected where the product of the ellipse axis lengths reaches a local maximum.

Harris Corners

- Originally developed as features for motion tracking
- Greatly reduces amount of computation compared to tracking every pixel
- Translation and rotation invariant (but not scale invariant)
Harris Corner in Matlab

% Harris Corner detector - by Kashif Shahzad
sigma=2; thresh=0.1; sze=11; disp=0;

% Derivative masks
dy = [-1 0 1; -1 0 1; -1 0 1];
dx = dy'; %dx is the transpose matrix of dy

% Ix and Iy are the horizontal and vertical edges of image
Ix = conv2(bw, dx, 'same');
Iy = conv2(bw, dy, 'same');

% Calculating the gradient of the image Ix and Iy
g = fspecial('gaussian',max(1,fix(6*sigma)), sigma);
Ix2 = conv2(Ix.^2, g, 'same'); % Smoothed squared image derivatives
Iy2 = conv2(Iy.^2, g, 'same');
Ixy = conv2(Ix.*Iy, g, 'same');

% My preferred measure according to research paper
cornerness = (Ix2.*Iy2 - Ixy.^2)./(Ix2 + Iy2 + eps);

% We should perform nonmaximal suppression and threshold
mx = ordfilt2(cornerness,sze^2,ones(sze)); % Grey-scale dilate
cornerness = (cornerness==mx)&(cornerness>thresh); % Find maxima
[rws,cols] = find(cornerness); % Find row,col coords.

clf ; imshow(bw); hold on;
p=[cols rws];
plot(p(:,1),p(:,2),'or');
title('bf Harris Corners')

Example (σ=0.1)
Example ($\sigma=0.01$)

Example ($\sigma=0.001$)
Harris: OpenCV Implementation

Template Matching
Problem: Features for Recognition

Templates

- Find an object in an image!

- Want Invariance!
  - Scaling
  - Rotation
  - Illumination
  - Deformation
Convolution with Templates

% read image
im = imread('bridge.jpg');
bw = double(im(:,:,1)) ./ 256;
imshow(bw)

% apply FFT
FFTim = fft2(bw);
imshow(FFTim)

% define a kernel
kernel=zeros(size(bw));
kernels(1, 1) = 1;
kernels(1, 2) = -1;
FFTkernels = fft2(kernel);

% apply the kernel and check out the result
FFTresult = FFTim .* FFTkernels;
result = real(ifft2(FFTresult));
imshow(result)

% select an image patch
patch = bw(221:240,351:370);
imshow(patch)

% select an image patch
patch = patch - (sum(sum(patch)) / size(patch,1) / size(patch, 2));

% define a kernel
kernel=zeros(size(bw));
kernels(1, 1) = 1;
kernels(1, 2) = -1;
FFTkernels = fft2(kernel);

% apply the kernel and check out the result
FFTresult = FFTim .* FFTkernels;
result = real(ifft2(FFTresult));
result = result ./ max(max(result));
result = (result ^ 1 > 0.5);
imshow(result)

% alternative convolution
imshow(conv2(bw, patch, 'same'))

Template Convolution
Recap: Convolution Theorem

\[ F(I \otimes g) = F(I) \cdot F(g) \]

\[ F(g(x, y))(u, v) = \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} g(x, y) \exp\{-i2\pi(ux + vy)\} \, dx \, dy \]
Convolution with Templates

% read image
im = imread('bridge.jpg');
bw = double(im(:,:,1)) ./ 256;;
imshow(bw)

% apply FFT
FFTim = fft2(bw);
bw2 = real(ifft2(FFTim));
imshow(bw2)

% define a kernel
kernel=zeros(size(bw));
kernel(1, 1) = 1;
kernel(1, 2) = -1;
FFTkernel = fft2(kernel);

% apply the kernel and check out the result
FFTresult = FFTim .* FFTkernel;
result = max(0, real(ifft2(FFTresult)));
result = result ./ max(max(result));
imshow(result)

% select an image patch
patch = bw(221:240,351:370);
imshow(patch)

% define a kernel
kernel=zeros(size(bw));
kernel(1:size(patch,1),1:size(patch,2)) = patch;
FFTkernel = fft2(kernel);

% apply the kernel and check out the result
FFTresult = FFTim .* FFTkernel;
result = max(0, real(ifft2(FFTresult)));
result = (result .^ 1 > 0.5);
imshow(result)

% alternative convolution
imshow(conv2(bw, patch, 'same'))

Convolution with Templates

• Invariances:
  • Scaling
  • Rotation
  • Illumination
  • Deformation
  • Provides
    • Good localization
Scale Invariance: Image Pyramid

Templates with Image Pyramid

- Invariances:
  - Scaling: Yes
  - Rotation: No
  - Illumination: No
  - Deformation: Maybe

- Provides:
  - Good localization: No
Template Matching, Commercial

Templates
Scale-invariant feature transform (SIFT)

Let’s Return to this Problem…
SIFT

• Invariances:
  • Scaling       Yes
  • Rotation      Yes
  • Illumination  Yes
  • Deformation   Maybe

• Provides
  • Good localization   Yes

SIFT Reference


SIFT = Scale Invariant Feature Transform
Invariant Local Features

- Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters.

Advantages of invariant local features

- **Locality**: features are local, so robust to occlusion and clutter (no prior segmentation)
- **Distinctiveness**: individual features can be matched to a large database of objects
- **Quantity**: many features can be generated for even small objects
- **Efficiency**: close to real-time performance
- **Extensibility**: can easily be extended to wide range of differing feature types, with each adding robustness
SIFT On-A-Slide

1. **Enforce invariance to scale:** Compute Gaussian difference max, for many different scales; non-maximum suppression, find local maxima: keypoint candidates

2. **Localizable corner:** For each maximum fit quadratic function. Compute center with sub-pixel accuracy by setting first derivative to zero.

3. **Eliminate edges:** Compute ratio of eigenvalues, drop keypoints for which this ratio is larger than a threshold.

4. **Enforce invariance to orientation:** Compute orientation, to achieve scale invariance, by finding the strongest second derivative direction in the smoothed image (possibly multiple orientations). Rotate patch so that orientation points up.

5. **Compute feature signature:** Compute a "gradient histogram" of the local image region in a 4x4 pixel region. Do this for 2x2 regions of that size. Orient so that largest gradient points up (possibly multiple solutions). Result: feature vector with 128 values (15 fields, 8 gradients).

6. **Enforce invariance to illumination change and camera saturation:** Normalize to unit length to increase invariance to illumination. Then threshold all gradients, to become invariant to camera saturation.

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**Step 1: Scale-space extrema detection**

1. **Enforce invariance to scale:** Compute Gaussian difference max, for many different scales; non-maximum suppression, find local maxima: keypoint candidates

2. **Localizable corner:** For each maximum fit quadratic function. Compute center with sub-pixel accuracy by setting first derivative to zero.

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Find Invariant Corners

1. **Enforce invariance to scale**: Compute Gaussian difference max, for many different scales; non-maximum suppression, find local maxima: keypoint candidates

   - Keypoints are detected using scale-space extrema in difference-of-Gaussian function $D$
   - $D$ definition:
     \[
     D(x, y, \sigma) = (G(x, y, k\sigma) - G(x, y, \sigma)) \ast I(x, y)
     \]
     \[
     = L(x, y, k\sigma) - L(x, y, \sigma)
     \]
   - Efficient to compute

Finding “Keypoints” (Corners)

Idea: Find Corners, but scale invariance

Approach:
- Run linear filter (diff of Gaussians)
- At different resolutions of image pyramid
Difference of Gaussians

surf(fspecial('gaussian',40,4))
surf(fspecial('gaussian',40,8))
surf(fspecial('gaussian',40,8) - fspecial('gaussian',40,4))
Find Corners with DiffOfGauss

```matlab
im = imread('bridge.jpg');
bw = double(im(:,:,1)) / 256;

for i = 1 : 10
    gaussD = fspecial('gaussian',40,2*i) - fspecial('gaussian',40,i);
    res = abs(conv2(bw, gaussD, 'same'));
    res = res / max(max(res));
    imshow(res); title(\['\bf i = ' num2str(i)\]); drawnow
end
```

Scale-space extrema detection

- Scale space construction
Gaussian Kernel Size $i=1$

Gaussian Kernel Size $i=2$
Gaussian Kernel Size $i=5$

Gaussian Kernel Size $i=6$
Gaussian Kernel Size $i=7$

Gaussian Kernel Size $i=8$
Gaussian Kernel Size $i=9$

Gaussian Kernel Size $i=10$
Key point localization

- Detect maxima and minima of difference-of-Gaussian in scale space
- Sample point is selected only if it is a minimum or a maximum of these points

Example of keypoint detection

(a) 233x189 image
(b) 832 DOG extrema
(c) 729 above threshold
SIFT On-A-Slide

Step 2: Key point localization

1. **Enforce invariance to scale:** Compute Gaussian difference max, for many different scales; non-maximum suppression, find local maxima: keypoint candidates

2. **Localizable corner:** For each maximum fit quadratic function. Compute center with sub-pixel accuracy by setting first derivative to zero.

3. **Eliminate edges:** Compute ratio of eigenvalues, drop keypoints for which this ratio is larger than a threshold.

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Localization

- 3D quadratic function is fit to the local sample points

- Start with Taylor expansion with sample point as the origin
  \[ D(X) = D + \frac{\partial D^T}{\partial X} X + \frac{1}{2} X^T \frac{\partial^2 D}{\partial X^2} X \]

  - where \( \hat{X} = -\frac{\partial^2 D^{-1} \partial D}{\partial X} \)

- Take the derivative with respect to \( X \), and set it to 0, giving
  \[ 0 = \frac{\partial D}{\partial X} + \frac{\partial^2 D}{\partial X^2} \hat{X} \]

- \( X = (x, y, \sigma)^T \) is the location of the keypoint
- This is a 3x3 linear system
Filtering

- Contrast (use prev. equation): \( D(\hat{X}) = D + \frac{1}{2} \frac{\partial D^T}{\partial X} \hat{X} \)
  - If \( |D(\hat{X})| < 0.03 \), throw it out

- Edginess:
  - Use ratio of principal curvatures to throw out poorly defined peaks
  - Curvatures come from Hessian:
  - Ratio of Trace(\(H\))^2 and Determinant(\(H\))
    \[ H = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{bmatrix} \]
    \[
    Tr(H) = D_{xx} + D_{yy}
    \]
    \[
    Det(H) = D_{xx}D_{yy} - (D_{xy})^2
    \]
  - If ratio > \((r+1)^2/r\), throw it out (SIFT uses \(r=10\))

Example of keypoint detection

(c) 729 left after peak value threshold (from 832)
(d) 536 left after testing ratio of principle curvatures
SIFT On-A-Slide

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Step 3: Orientation assignment

- Create histogram of local gradient directions computed at selected scale
- Assign canonical orientation at peak of smoothed histogram
- Each key specifies stable 2D coordinates (x, y, scale, orientation)
**SIFT On-A-Slide**  
**Step 4: Generation of key point descriptors**

1. **Enforce invariance to scale:** Compute Gaussian difference max, for many different scales; non-maximum suppression, find local maxima: keypoint candidates
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**SIFT vector formation**

- Thresholded image gradients are sampled over 16x16 array of locations in scale space
- Create array of orientation histograms
- 8 orientations x 4x4 histogram array = 128 dimensions
Nearest-neighbor matching to feature database

• Hypotheses are generated by **approximate nearest neighbor** matching of each feature to vectors in the database
  • SIFT use best-bin-first (Beis & Lowe, 97) modification to k-d tree algorithm
  • Use heap data structure to identify bins in order by their distance from query point

• **Result:** Can give speedup by factor of 1000 while finding nearest neighbor (of interest) 95% of the time

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3D Object Recognition

• Extract outlines with background subtraction
3D Object Recognition

• Only 3 keys are needed for recognition, so extra keys provide robustness
• Affine model is no longer as accurate

Recognition under occlusion
Test of illumination invariance

• Same image under differing illumination

Examples of view interpolation
Location recognition

SIFT

- Invariances:
  - Scaling: Yes
  - Rotation: Yes
  - Illumination: Yes
  - Deformation: Maybe

- Provides:
  - Good localization: Yes
Questions?