

Foundations Qualifying Examination, Spring 2005

This exam is closed book. Complete as many problems as you can. Justify all your answers unless instructed otherwise. You may do the problems in any order, but start each problem on a new page and label the problem. Show your work, as partial credit may be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. Be neat.

Part I: Formal Languages and Automata (50 points)

1. Let $G = (V, \Sigma, R, S)$ be a grammar.
 - (a) (6 points) Define precisely what it means for G to be in Chomsky Normal Form.
 - (b) (5 points) Explain why Chomsky Normal Form is useful by stating one result concerning it (a theorem, lemma, etc.) and describing how this result can be applied to solve some other problem. You do not have to prove the result itself.
2. Let $L = \{ a^n w w a^m \mid w \in \{b, c\}^* \text{ and } m, n \geq 0 \text{ and } m + n > 2 \}$.
 - (a) (7 points) State a version of the pumping lemma for regular languages.
 - (b) (7 points) Prove that L is not regular using your stated pumping lemma.
3. For each of the languages below, state the *smallest* class that is a member of, choosing among the following options. No proof is necessary. However, a word of explanation will help communicate your thoughts and may lead to partial credit for incorrect answers. The classes, in increasing order of size according to \subseteq , are:
 - class of finite languages
 - class of regular languages
 - class of context-free languages
 - class of Turing-decidable languages
 - class of Turing-acceptable languages
 - class of all languages

(5 points each)

- (a) $L_1 = \{ xyz\#y^R \mid y \in \{a, b\}^* \text{ and } x, z \in \{0, 1\}^* \}$
- (b) $L_2 = \{ \langle M \rangle \mid M \text{ is a DTM and } aba \in L(M) \}$.
- (c) $L_3 = \{ x \in \{0, 1\}^+ \mid x \equiv 2 \pmod{3} \}$. Here x is to be interpreted as a binary number, so that $0101 \in L_3$ and $110 \notin L_3$.
- (d) $L_4 = \{ x\#x^R\#x \mid x \in \{a, b\}^* \}$ (note x appears three times)
- (e) $L_5 = \{ x \in \{A \dots Z, |, a \dots z, \varepsilon, \rightarrow, \backslash \mathbf{n}\}^* \mid x \text{ is a context free grammar} \}$. Example element of L_5 : " $A \rightarrow aAB \mid \varepsilon \backslash \mathbf{n} B \rightarrow b$ ", where $\backslash \mathbf{n}$ is the newline character

Part II: Computability and Complexity (50 points)

4. (12 points) Let A be an infinite set. Show that A is Turing-acceptable if and only if A is Turing-enumerable.

Recall that a set A is *Turing-acceptable* if there is a Turing machine M such that $A = L(M) = \{x \mid M \text{ on } x \text{ halts}\}$. An infinite set A is *Turing-enumerable* if there exists a two-tape DTM M , whose second tape is a one-way write-only tape, such that when it is given the empty string ϵ as the input, M prints an infinite sequence of strings x_1, x_2, \dots on the second tape, with every two strings separated by a blank, such that $A = \{x_1, x_2, \dots\}$.

5. (12 points) Consider the following set

$$S = \{\langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are DTMs, } L(M_1) \text{ is regular but } L(M_2) \text{ is not regular}\}.$$

Is S recursively enumerable? Justify your answer.

6. (12 points) Let $A \in NSPACE(n)$. Construct a DTM M such that M accepts A in $O(n^2)$ space. (Note: No credit will be given if you simply apply Savitch's Theorem without giving a construction of M .) Justify your answer.
7. (14 points) Instances of k SAT are k -CNF formulas. That is, each clause C in a k -CNF formula consists of exactly k different literals. Moreover, if C contains a literal ℓ , then its complement $\neg\ell$ does not occur in C . We want to determine whether a given k SAT instance is satisfiable. Given the fact that 3SAT is NP-complete, show that for any $k > 3$, k SAT is also NP-complete.