Qualifying Examination

This exam is closed book. Complete as many problems as you can, and justify your answers. All problems are equally weighted. You may do the problems in any order, but start each problem on a new page and label the problem. Plan your time wisely and do as many problems as possible. Show your work, as partial credit may be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. Be neat. If you need more space, use the back of the paper.

Problem	Points	Grade
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	**100**	

1. Construct a DFA to accept the language of all binary strings that have 101 as a substring.

2. Consider the following ϵ-NFA:

δ	ϵ	a	b	c
$\rightarrow p$	$\{q\}$	$\{p\}$	$\{r\}$	$\emptyset$
q	$\{r\}$	$\{p\}$	$\{r\}$	$\{p, q\}$
$*r$	$\emptyset$	$\{r\}$	$\{r\}$	$\{r\}$

   (a) Derive a regular expression for the language accepted by this ϵ-NFA. (b) Convert this ϵ-NFA to a DFA.

3. Let L be a language defined over binary alphabet $\{0, 1\}$, where

$$L = \{w : \text{the number of 1's in } w \text{ is twice as many as the number of 0's in } w\}.$$

   Is L regular? Is L context-free? Justify your answers.

4. Let G be a CFG in Chomsky normal form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly how many steps are needed for any derivation of w?

5. Suppose L_1 is regular and L_2 is context-free, construct a PDA M to accept $L_1 \cap L_2$. Note: You need to provide the transition function of M.

6. Let L be a language. Define an operation MIN as follows:

$$\text{MIN}(L) = \{w \in L : w \text{ does not have a proper prefix in } L\}.$$

   Show that there exists a context-free language L such that $\text{MIN}(L)$ is not context-free.

7. Let N be a one-tape nondeterministic Turing machine. Construct a one-tape deterministic Turing machine to simulate N.

8. Let L be a language that can be accepted by a Turing machine. Show that L can be enumerated by a Turing machine; that is, construct a deterministic Turing machine M such that it takes no inputs and prints out exactly all elements in L.

9. Let $L_u = \{\langle M, w\rangle : M \text{ is a TM and } M \text{ accepts } w\}$ and we know that L_u is not decidable. Let $RL = \{\langle M\rangle : M \text{ is a TM and } L(M) \text{ is a regular language}\}$. Show that RL is not decidable by constructing a reduction from L_u to RL.

10. State and prove Rice Theorem. Let $L = \{\langle M\rangle : M \text{ is a PDA and } L(M) = \emptyset\}$. Can Rice Theorem be applied to show that L is not decidable? Justify your answer.