

Foundations Qualifying Examination, Fall 2003

This exam is closed book. All problems are equally weighted. Complete as many problems as you can. Justify all your answers. You may do the problems in any order, but start each problem on a new page and label the problem. Show your work, as partial credit may be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. Be neat.

Problem	Points	Grade
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Student Name: _____

ID: _____

- Let $L = \{0^i 1^j 0^k \mid i, j, k \geq 0 \text{ and } i + k = j\}$.
 - Is L regular? If so, construct a DFA that accepts L . If not, prove that L is not regular by showing that $\text{Index}(R_L) = \infty$, where $xR_L y$ iff $(\forall w)[xw \in L \leftrightarrow yw \in L]$ and $\text{Index}(R_L)$ is the number of equivalence classes of R_L .
 - Is L context-free? If so, construct a context-free grammar to generate L . If not, provide a proof that L is not context-free.
- Prove that context-free languages are not closed under intersection.
- Prove that deciding whether two given Turing machines accept the same language is not Turing-acceptable. (Note: you may assume that the halting problem is not Turing-decidable without providing a proof.)
- Let M be a $T(n)$ -time bounded one-tape NTM, where $T(n) \geq n$. Construct a one-tape DTM M' to simulate M in time $O(d^{T(n)})$, where $d > 1$ is a constant depending only on M .
- Prove that the following language is NP -complete.

$$K = \{\langle M, x, n \rangle \mid M \text{ is an NTM and accepts } x \text{ in } |n| \text{ steps}\}$$