Foundations Qualifying Examination

This exam is closed book. All problems are equally weighted. Complete as many problems as you can. Justify all your answers. You may do the problems in any order, but start each problem on a new page and label the problem. Show your work, as partial credit may be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. Be neat. If you need more space, use the back of the paper.

Problem	Points	Grade
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	**100**	

1. Construct a DFA to accept the language of all binary expansions of positive integers that are congruent to zero mod 4.
2. Describe an algorithm that converts an ϵ-NFA to a DFA.
3. Given a DFA, describe how to obtain an equivalent DFA with minimum number of states.
4. Give a language that is context-free, but is not regular. Justify your answer.
5. Show that pushdown automata with two stacks are strictly more powerful than pushdown automata with only one stack.
6. What is Halting Problem? Use diagonalization to show that Halting Problem is not decidable.
7. Let $K = \{\langle M \rangle : M \text{ is a TM and } M \text{ on input } \langle M \rangle \text{ halts}\}$. Show that K is not decidable by reducing Halting Problem to K.
8. Let L be a language accepted by a Turing machine. Show that L can be recursively enumerated by a deterministic Turing machine.
9. Show that nondeterministic Turing machines can be simulated by deterministic Turing machines.
10. Let $L = \{\langle M \rangle : M \text{ is a TM and } L(M) = \emptyset\}$. Show that L is not decidable without using Rice Theorem. You may, however, use the proof technique of Rice Theorem.