A Shifting Strategy for Dynamic Channel Assignment

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1. Introduction and Demand Model

In this work, a new algorithm for dynamic channel assignment (DCA) in cellular networks is presented. The ability to adaptively distribute channel resources in response to heterogeneous demand and interference, while maintaining spectral efficiency, are important considerations for multimedia wireless transmission. DCA techniques may be broadly classified as graph-theoretic, mathematical programming and meta-heuristic search, or some combination thereof. Mathematical programming approaches to DCA [1] consider the minimization of a cost function such as the allocated bandwidth under the constraint that channel reuse takes place above specified interference levels. Many DCA problems are NP-hard. The engineering approaches have been based on polling available channels and selecting the first channel that can transmit above the interference threshold or based on search and borrow of unused channels from neighboring cells [2]. Although various algorithmic models and solution techniques have been proposed for DCA, the performance of such algorithms in the context of time and spatially varying traffic demand has been examined to a lesser extent.

The focus of this work is to design techniques for an NP-hard DCA problem and analyze their performance in the presence of random spatial and time-varying demand processes. Here, cells that generate variable demands in time are denoted as Type_v cells. They are randomly distributed in a square cellular grid of size 7 × 7. The distribution of Type_v cells in space is characterized by a Bernoulli distribution with probability p_v governing the occurrence of a Type_v cell in n_c = 49 cells. The choice of p_v for a fixed n_c determines the average number of Type_v cells that populate the network. Here p_v = 0.2. Fig. 1 shows nine spatial distributions for which the proposed DCA algorithm is evaluated.

Each Type_v cell demands channels in time according to a two state (on-off) discrete time Markov chain. In the on state, the channel demand is set to one, and in the off state the demand is zero. The specified steady state probability of being on is used to determine the Markov transition probability matrix and the temporal dynamics of the channel demand function. In this work, the channel assignment algorithm first determines the assignments for all possible combinations of on-off states in a given spatial distribution containing n_v cells of Type_v. Then the probability mass function (PMF) of k channels required is obtained by weighting each on-off state configuration that required k channels with its respective binomial probability Pr[N_{on} = i, n_v], where N_{on} is the number of Type_v cells that are on and n_v is the number of Type_v cells in the cellular system. Therefore,

\[ Pr[Ch_{req} = k] = \sum_{i=k}^{n_v} Pr[Ch_{req} = k | N_{on} = i] Pr[N_{on} = i]. \]

The proposed DCA algorithm that utilizes a shifting strategy is presented next.

![Figure 1: Stochastic distribution of variable demand (Type_v) cells (shaded). p_v = 0.2.](image)
2. Shifting Strategy

The new DCA method uses a combination of Integer Programming (IP) and heuristics to minimize the number of channels required to satisfy traffic demand. A threshold criterion based on the carrier-to-interference ratio determines the minimum channel reuse distance \( r \). The problem is viewed as a collection of \( (r + 1) \times (r + 1) \) sized subproblems. The optimal number of channels for a subproblem can be found in a practical amount of execution time when \( r \) is small, and it forms a lower bound on the number of channels for the full problem. A core IP (CIP) model enforces demand satisfaction and cumulative cochannel interference constraints; this is formulated and solved for each subproblem. For example, Fig. 2(a) shows an assignment for distribution \( g \) in Fig. 1 for \( r = 2 \) and the numbers denote channel indices.

An upper bound for the full problem is created from the subproblem solutions using a shifting strategy: SHIFT-IP. SHIFT-IP seeks an optimal solution for the full problem, where that solution has special structure. SHIFT-IP relies on the observation that each cell group’s solution determines a family of isomorphic solutions. Let \( 0, 1, \ldots, f_{\text{max}} - 1 \) be the available channel numbers. From any group’s solution another solution can be created by replacing each assignment of a channel \( f \) with an assignment of channel \((f + f') \mod f_{\text{max}} \) for some shift integer \( f' \). See Fig. 2(b) as derived from shifting the solution in Fig. 2(a). SHIFT-IP formulates CIP for the entire cellular system, then adds shift variables and shift constraints so that each subproblem’s solution will be a shift of a subproblem’s optimal solution. SHIFT-IP calls the IP solver on this augmented formulation. If it finds an optimal solution that matches the largest lower bound obtained from subproblems, then SHIFT-IP’s solution is provably optimal for the full cellular system. Otherwise, a second approach, GREEDY-IP, is invoked. GREEDY-IP uses the CIP formulation iteratively by augmenting local solutions to an ordered list of ascending demand values.

![Figure 2: (a) Local solutions in 3 x 3 sized grids. (b) Assembling a solution using shifts of solutions derived in (a). Groups A, B, C and D have shifts 2, 0, 1, 2, respectively.](image)

3. Results and Conclusion

The channel assignment approach discussed briefly above is run for each distribution in Fig. 1. For each case, assignments are obtained for all possible on-off configurations of the Type-\( e \) cells. For the spatial distributions selected, there are 256 to 8192 unique states of the network. Table 1 shows SHIFT-IP optimality results. For almost all configurations of Fig. 1 SHIFT-IP obtains provably optimal results when all Type-\( e \) cells are in the on state. It also obtains optimal results for a large percentage of demand states for each configuration; the average across all spatial configurations is 85.52%. However, it is not always the case that when SHIFT-IP achieves optimal results for all Type-\( e \) cells in the on state it also reaches optimality for 100% of the demand states.

Figs. 3(a-i) compare the PMFs of required channels obtained from the IP-based channel assignment strategy with a non-IP based SEQ-GREEDY algorithm. The PMFs are calculated as discussed in Section 1. The SEQ-GREEDY method performs a sequential allocation of channels by traversing the channels in a specified order and assigning the first available channel (from an ordered list) that satisfies demand and channel constraints. SEQ-GREEDY performs nearly as well as IP for configurations \( a, b, c, e, f, g \) and departs considerably from the optimal results obtained by IP for cases \( d, h, i \). Differences come from the ability of the IP-based heuristic to perform both local and constrained global optimization. SEQ-GREEDY sometimes benefits from fortuitous channel
assignments. This allows it to sometimes match the IP performance in cases in which Type_v cell groups are large and/or densely packed (e.g. b, c, g, f).

The SHIFT-IP algorithm finds optimal channel assignments 75% to 100% of the time on different spatial distributions of channel demand with consistently fast execution times. SHIFT-IP’s effectiveness diminishes with increase in the number of Type_v cells. This is partly due to the NP-hard nature of this DCA problem. When the subproblems are well separated in space the IP strategy quickly converges, assigning a globally optimal solution. When the cumulative interference dominates over the reuse distance, the IP solutions require fewer channels than given by the non-IP algorithm. As the density of subproblems increase, however, the IP approach yields fewer optimal answers than for sparse ones. Some improvement in effectiveness can be achieved by randomly permuting the list of channel numbers used by a subproblem’s assignments before creating shift variables and constraints. This allows SHIFT-IP to explore a broader set of isomorphic solutions. SHIFT-IP addresses a limitation of previous DCA work that used a cluster-replication heuristic to provide an upper bound for an IP model [3].

<table>
<thead>
<tr>
<th>Fig. 1</th>
<th>Config.</th>
<th>% Times SHIFT-IP Probably Optimal</th>
<th>SHIFT-IP Probably Optimal for all Type_v Cells On?</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>72.55</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>100</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>92.96</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>74.4</td>
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<td></td>
</tr>
<tr>
<td>e</td>
<td>87.45</td>
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<td></td>
</tr>
<tr>
<td>f</td>
<td>81.72</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>82.85</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>h</td>
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</tr>
<tr>
<td>i</td>
<td>88.28</td>
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<td></td>
</tr>
</tbody>
</table>

Table 1: SHIFT-IP optimality results when minimum reuse distance r = 2.

Figure 3: PMF of channels required when minimum reuse distance r = 2 for distributions in Fig. 1. Probability a cell is on = 0.57. (Circular dots:IP-based results) (Rectangles:SEQ-GREEDY algorithm).

References

