Pricing and Revenue Sharing Mechanism for Secondary Re-distribution of Data Service for Mobile Devices

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Abstract — Cellular Network Providers (CNP) provide users with wireless data access to meet the growing ubiquitous demand for the Internet. As users subscribe to a fixed data plan for a monthly flat fee, some users may exhaust their data allowance before the end of the billing cycle, while others underutilize their monthly quota. To take advantage of such underutilization, Khausik et. al. propose a mechanism for *ad hoc* bandwidth redistribution that allows subscribers to sell their unused bandwidth to users needing Internet access in exchange for some financial compensation as and when opportunities arise. There exists a popular belief that allowing such on-demand ad hoc service is not beneficial to CNP. This paper seeks to address and counter this opinion by proposing a pricing scheme and a revenue sharing mechanism that makes the provision of ad hoc connection *advantageous to CNP.* Our revenue sharing mechanism provides economic incentives to CNP. The simulation results show that our revenue sharing model ensures that CNP receives the majority portion of the revenue gained, regardless of the amount. Secondly, our pricing model ensures traffic from ad hoc users has minimal impact on the connection quality of current subscribers. In this model, we use Shapley value as the basis for deriving the revenue sharing.

Index Terms **- Network Pricing, Wireless Network, data sharing.**

I. INTRODUCTION

Cellular Network Providers (CNP) sell monthly data plan to *subscribers*. Some subscribers exhaust their data quota before the end of the monthly cycle and are left with no access to internet; then, there are others who underutilize and have no choice but to "scrap" their remaining data allowance. In addition, people traveling outside of their countries may purchase prepaid data plan but may not be able to use up the quota before the end of the trip. To meet the need for on-demand connection, Kaushik et. al. propose a framework for mobile ad-hoc network service, *CollabAssure* [6] to allow data plan subscribers to sell bandwidth to other users (*ad hoc buyers*) for monetary rewards. We term these subscribers who sell their unused bandwidth as *resellers*. However, many have argued that such arrangement provides very little incentive for CNP, and at the same time causes some challenges. For example, allowing subscribers to be resellers means there will be more parties providing network connection, making it more challenging for CNP to have an oversight of the market or control over pricing, and may lead to market saturation. From network management perspective, providing service to ad hoc users may cause the market for Internet access to become saturated. As CNP is ultimately responsible for the additional traffic generated by ad hoc users, this may adversely affect the quality of service (QoS) of regular subscribers. More than attending to these concerns, our proposed pricing and revenue sharing mechanisms provide revenue incentives to CNP to implement the provision of ad hoc service via subscribers.

Most literatures on ad hoc network service focus on the technical means of the provision of such service. There has been little discussion on the aspects of pricing and revenue. For example, Alvarion [1] and Cisco [2] provide Wi-Fi service incorporated into 3G and LTE service. Alternatively, [3] proposes a methodology that uses ad hoc nodes to forward data from their provider to their neighbors, and [4] proposes data dissemination through Wi-Fi when network is congested. Similarly in [5], the authors propose an application, *"SmartParcel"*, to allow users to collaboratively share the meta data. For example, instead of downloading the data from a content provider (such as BBC, google, etc), a user may download the same data from a nearby smart phone. However, none of these papers address the financial concerns of ad hoc connection services. In addition, these literatures do not address the critical concerns of CNP, such as market saturation, revenue sharing, and the impact on the current subscribers' connection quality. Bearing a degree of resemblance to the focus of our paper is a proposal in [15] where the authors suggest pricing maximization and revenue sharing mechanism for global Wi-Fi provider (*Skype*) and local Wi-Fi providers to make profits. Our proposal, on the hand, is not intended to maximize profits for CNP or resellers (subscribers); rather, the objective is to provide an incentive to CNP to support ad hoc service while giving an acceptable compensation to resellers. This paper includes a pricing model and a revenue sharing mechanism which collectively address CNP's concerns about traffic load, subscribers' connection quality, and market and price oversight. Importantly, our proposed mechanisms are also designed to manage subscribers' intensity of reselling transactions based on the amount of unused bandwidth available in their data plan at the point of each transaction. Last but not least, the revenue sharing mechanism assures economic incentives for CNP.

Our pricing model is built on two principles. The first is dynamic pricing which fluctuates according to the level of bandwidth demand. To set the minimum sale price, CNP must consider the overall traffic load such that the connection quality of regular subscribers is not compromised when traffic from ad hoc buyers is attended to. This means, CNP has to consider the overall bandwidth demand from both subscribers and ad hoc users. Similarly, reseller computes his/her price to ad hoc buyers according to level of ad hoc demand –this price increases as the level of demand increases. The second principle is, the final price charged to ad hoc buyer must be equal or higher than the minimum price set by CNP. This is because the minimum sale price set by CNP considers the overall bandwidth demand on the network, not just the demand from ad hoc buyers. Hence, when the price set by CNP is higher than the price set by reseller, the former prevails. Importantly, our study shows that this pricing model guarantees a minimal impact on the connection quality of current subscribers. These pricing frameworks are formulated into Network Utility Maximization (NUM) problem [11], which is resolved using subgradient based algorithm.

Our investigation on revenue sharing begins by considering two different scenarios: First, a *cooperative* scenario where the revenue from the resale of the bandwidth is shared between CNP and the reseller. We build our cooperative revenue sharing model based on *Shapley value* mechanism [17] because of its capacity to divide the revenue "fairly" between parties involved. Second, a *non-cooperative* scenario where the CNP sets and keeps the minimum sale price, while the reseller sets the final sale price to the ad hoc buyer and gets to keep the difference between the final sale price and the minimum sale price. Our study shows that the cooperative scenario provides more incentives to CNP to support ad hoc internet access. That is, our cooperative revenue sharing model ensures that CNP receives the majority portion of the revenue gained, regardless of the amount of the final sale price set by the reseller. Our model also prevents subscribers from achieving profitability (after deducting their monthly subscription fee) through reselling transactions to prevent market saturation. Furthermore, our simulation shows that this model discourages resellers from selling their bandwidth during peak hours and provides the upper bound of the percentage a reseller may receive. These confirm that our revenue sharing model is advantageous to CNP, while achieving "fair" compensation to resellers.

The rest of this paper is organized as follows. We begin by formulating the problem in section II where we review the architecture and the impact of reselling bandwidth on network. In section III, we propose the pricing mechanism between the reseller and the ad hoc buyer, and the CNP and the reseller. Following that, we present the revenue sharing mechanism in section IV. Here, we compare the revenue sharing in both cooperative and non-cooperative scenarios. The simulation results are presented in section V, followed by concluding remarks.

II. Problem Formulation

A. Architecture Review

In [6], Kaushik et. al. propose a framework for users without data plan to get mobile ad-hoc network service by buying connection from physically neighboring users with data subscription. In other words, this is a framework for subscribers to be resellers. Designed and implemented for Android mobile devices, to get network access, an ad hoc buyer sends his/her request to nearby reseller(s). The request is delivered using technologies that allow ad hoc connection, such as Wi-Fi or BlueTooth. Next, the resellers present their price per unit time to the prospect buyer. Then, ad hoc buyer chooses the reseller based on the price offered and connects to the Internet through the selected reseller. Here, the reseller is acting as a redistributor of bandwidth, and a mediator between the ad hoc buyer and CNP.

B. Impact on CNP

In order to design an appropriate pricing mechanism, the impact of additional traffic generated from catering to ad hoc network service requests must be considered. The study in [6] shows that most of the bandwidth redistribution in US occurs during peak hours. With this additional traffic from ad hoc users during peak hours, network may become even more congested, increasing the burden on CNP. Moreover, higher traffic load may also negatively affect the connection quality of the subscribers and degrade network performance, especially during peak hours. From a financial perspective, while ad hoc buyers may enjoy the service and resellers may receive some monetary gain from the sales, CNP may have to incur higher costs in order to support the additional traffic without financial benefits. In addition, if ad hoc network access becomes widely available at low prices, subscribers with low data usage may potentially cease their subscription and switch to ad hoc service. Higher demand for ad hoc connection in turn encourages some users to subscribe to data plan with the intention to resell it for profit. Over time, this may result in market saturation and loss of more revenue for CNP. Given all these concerns and the potential ripple effects, the general opinion is that there is very little incentive for CNP to allow their subscribers to sell their bandwidth to take advantage of data plan underutilization.

Pertaining to the concerns and challenges raised, we propose a pricing and revenue sharing strategy that not only assure economic incentives for CNP, but also discourage subscribers from selling their bandwidths for profit. The pricing mechanism is made up of two parts: First, the pricing strategy between CNP and resellers, where CNP determines the minimum price to resell the bandwidth at. At the same time, resellers compute and decide their final sale price to the prospective ad hoc buyers based on the level of demand. Following that, we address the revenue sharing mechanism between resellers and CNP in both cooperative and non-cooperative scenarios.

III. Pricing Mechanism

In this section, we address the two parts of the pricing mechanism. Figure 1 below illustrates the overview of the transaction: after an ad hoc buyer $\bar{s}$ makes his/her the request for connection, CNP presents the reseller $\dot{s}$ with the *minimum resell price* $g_{\bar{s}}$.At the same time, reseller computes the *reseller's price* $\lambda_{\dot{s}}$, and determine the *final sale price* $\lambda_{\bar{s}}$, such that $\lambda_{\bar{s}} = \max(\lambda_{\dot{s}}, g_{\bar{s}})$, and presents $\lambda_{\bar{s}}$ to the ad hoc buyer $\bar{s}$. The buyer $\bar{s}$ pays the reseller at price $\lambda_{\bar{s}}$. The pricing mechanism also considers the presence of multiple resellers and multiple ad hoc buyers at any point of time. We begin by first addressing reseller's price to ad hoc buyer.

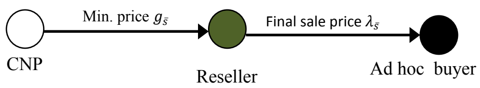

Fig. 1. Pricing mechanism.

A. Reseller's Price

Let $\dot{S}$ denotes a set of resellers and $\bar{S}$ denotes a set of *ad hoc buyers*, for $\dot{s} \in \dot{S}$ and $\bar{s} \in \bar{S}$ respectively. The objective of ad hoc buyer $\bar{s}$ is to solve

$$\max\ U_{\bar{s}}(x_{\bar{s}}, \lambda_{\bar{s}}^{t}), \quad \text{for } x_{\bar{s}}, \lambda_{\bar{s}}^{t} \geq 0, \tag{1}$$

where $x_{\bar{s}}$ denotes the amount of data usage by ad hoc buyer $\bar{s}$ and $\lambda_{\bar{s}}^{t}$ denotes the price to be paid by $\bar{s}$ for the network access at time t. The price is dynamically determined according to the level of demand for ad hoc service at time t. The utility function of the ad hoc buyer is defined as follows.

$$U_{\bar{s}}(x_{\bar{s}}, \lambda_{\bar{s}}^{t}) = U_{bw}(x_{\bar{s}}) + U_{cost}(x_{\bar{s}}, \lambda_{\bar{s}}^{t}), \tag{2}$$

where $U_{bw}(x_{\bar{s}})$ and $U_{cost}(x_{\bar{s}}, \lambda_{\bar{s}}^{t})$ denote user $\bar{s}$ utility with bandwidth consumption $x_{\bar{s}}$ and service cost, respectively. Utility function $U_{cost}(x_{\bar{s}}, \lambda_{\bar{s}}^{t})$ is defined as follows.

$$U_{cost}(x_{\bar{s}}, \lambda_{\bar{s}}^{t}) = 1 - \frac{x_{\bar{s}}\ \lambda_{\bar{s}}^{t}}{m_{\bar{s}}},$$

where, $m_{\bar{s}}$ denotes the budget that buyer $\bar{s}$ is willing to spend for bandwidth $x_{\bar{s}}$; $(x_{\bar{s}}\ \lambda_{\bar{s}}^{t})$ can be interpreted as the price that ad hoc user must pay for the service. Thus, ideally, user's budget matches the price that user must pay for the service, such that $U_{cost}(x_{\bar{s}}, \lambda_{\bar{s}}^{t}) = 0$, that is when $\frac{x_{\bar{s}}\ \lambda_{\bar{s}}^{t}}{m_{\bar{s}}} = 1$. Therefore, given price $\lambda_{\bar{s}}^{t}$, ad hoc buyer uses $m_{\bar{s}}$ to influence the amount bandwidth $x_{\bar{s}}$ allocated to him/her. Considering the mobile of reseller $\dot{s}$ is operating at frequency band $W_{\dot{s}}$, the utility function of bandwidth usage is defined as follows

$$U_{bw}(x_{\bar{s}}) = W_{\dot{s}} \log\left(x_{\bar{s}}\left(1 + \frac{P_{\dot{s}}\ |c_{\dot{s}}|^{2}}{\partial_{\dot{s}}^{2}\ W_{\dot{s}}}\right)\right),$$

where $P_{\dot{s}}$ is the transmission power of user $\bar{s}$ mobile device, $c_{\dot{s}}$ is the channel gain from seller $\dot{s}$ to user $\bar{s}$, and $\partial_{\dot{s}}^{2}$ is the Gaussian noise variance for the channel between $\dot{s}$ and s [13]. In other words, $U_{bw}(x_{\bar{s}})$ is influenced by the channel quality and amount of bandwidth.

The objective of reseller $\dot{s}$ is to maximize his/her own revenue without exceeding his/her monthly bandwidth capacity. The maximization problem is expressed as follows.

$$\max \sum_{\bar{s}\epsilon\dot{s}} x_{\bar{s}} \cdot \lambda_{\bar{s}}^{t}$$
$$s.t. \quad \sum_{\bar{s}\epsilon\dot{s}} x_{\bar{s}} \leq C_{\dot{s}}\ ,$$
$$over\ x_{\bar{s}} \geq \bar{0},$$

where capacity $C_{\dot{s}}$ is amount of bandwidth that reseller is selling and the aggregation of $C_{\dot{s}}$ does not exceed the monthly threshold of his/her data plan. Considering the respective objectives of ad hoc buyer and reseller, the problem can be formulated into network utility maximization (NUM) proposed by Frank Kelly in [11].

$$max \sum_{\bar{s}\epsilon\dot{s}} U_{\bar{s}}(x_{\bar{s}}, \lambda_{\bar{s}}^{t}) \tag{3}$$
$$s.t. \quad \sum_{\bar{s}\epsilon\dot{s}} x_{\bar{s}} \leq C_{\dot{s}}\ ,$$
$$\text{over } x_{\bar{s}}, \lambda_{\bar{s}}^{t} \geq 0.\ \forall \dot{s}, \bar{s},$$

The Lagrangian optimization problem is formulated as $L(x, \lambda) = \sum_{\bar{s}\epsilon\dot{s}} U_{\bar{s}}(x_{\bar{s}}, \lambda_{\dot{s}}) - \sum_{\bar{s}\epsilon\dot{s}} \lambda_{\dot{s}}\ x_{\bar{s}} + \lambda_{\dot{s}}\ C_{\dot{s}}$, where $L(.)$ is the Lagrangian form and λ is known as a set of Lagrangian multipliers, which is often interpreted as the link price, and x is a vector of $x_{\bar{s}}$, for $\forall \bar{s}\epsilon\dot{s}$.

The common solution to NUM problem is the subgradient based method [12]. Typically, the dual problem D to the primal problem of (3) is constructed as follows

$$\min D(\lambda_{\dot{s}}), \quad s.t \quad \lambda_{\dot{s}} \geq 0, \tag{4}$$

where the dual function

$$D(\lambda_{\dot{s}}) = \max_{0 \leq x \leq x^{max}} L(x, \lambda_{\dot{s}}).$$

To solve $D(\lambda_{\dot{s}})$, first maximize over $x_{\bar{s}}$ given $\lambda_{\dot{s}}$. That is

$$x_{\bar{s}} = \underset{0\ \leq\ x_{\bar{s}}\ \leq\ x_{\bar{s}}^{max}}{\arg\max} \left(U_{\bar{s}}(x_{\bar{s}}, \lambda_{\dot{s}})\right). \tag{5}$$

Next, $L(x, \lambda_{\dot{s}})$ is minimized with subgradient projection method in an iterative solution given by

$$\lambda_{\dot{s}}^{t+1} = \left[\lambda_{\dot{s}}^{t} - \sigma^{t}\left(C_{\dot{s}} - \sum_{\bar{s}\epsilon\dot{s}} x_{\bar{s}}\right)\right]^{+}, \tag{6}$$

where $C_{\dot{s}} - \sum_{\bar{s}\epsilon\dot{s}} x_{\bar{s}}$ is a subgradient of $D(\lambda_{\dot{s}})$ and σ^{t} denote the step size to control the tradeoff between a convergence guarantee and the convergence speed, such that

$$\sigma^{t} \rightarrow 0, as\ t \rightarrow \infty\ and \sum_{t=1}^{\infty} \sigma^{t} = \infty. \tag{7}$$

Generally, the subgradient based solution relies on feedback loop mechanism. That is, the ad hoc buyer determines the transmission rate according to the price set by the reseller by solving (5) and the price is adjusted according to the traffic load by solving (6). It is repeated until it converges to an optimal solution. Price $\lambda_{\dot{s}}$ is also indication of the demand for ad hoc service. However, before determining the final sale price, reseller must consider the minimum price charged by CNP. It is because reseller depends on CNP's infrastructure to provide the service. The discussion on minimum price is addressed in the next section.

Proposition 1: If the step size σ in (6) satisfies (7), then the subgradient based algorithm converges to the optimal solution of problem (3). [7,11] ■

B. Minimum Resale Price by CNP

Here, we address how CNP determines the minimum resale price of bandwidth. Consider a network managed by CNP with a set of links L, and a set of link capacities C over the links in L. Given a utility function $U_{s}(x_{s}, \lambda_{s})$ of data plan subscriber s (a.k.a reseller) with bandwidth usage of x_{s} and additional traffic generated by ad hoc buyers in $\bar{S}$, the maximization problem can be formulated as follows.

$$max \sum_{s\in S} U_{s}(x_{s}) + \sum_{\dot{s}\in\dot{S}} \sum_{\bar{s}\epsilon\dot{s}} U_{\bar{s}}(x_{\bar{s}}), \tag{8}$$

$$s.t. \sum_{\bar{s} \epsilon \dot{s}} x_{\bar{s}} \le C_{\dot{s}} , \quad (8.a)$$

$$\sum_{s \in l} x_s + \sum_{\bar{s} \in l} x_{\bar{s}} \le C_l, \quad (8.b)$$

$$x'_{s,l} = x_{s,l} , \quad (8.c)$$

$$\text{over } x_{\bar{s}}, x_s \ge 0,$$

where S is a set of subscribers, for $\dot{S} \subseteq S$, $s \in S$, $\bar{s} \in \bar{S}$. $\bar{s} \in l$ and $s \in l$ denotes subscriber s and ad hoc buyer $\bar{s}$ who are transmitting data through link l. Conditions (8.a) and (8.b) guarantee the link is *feasible*, i.e. the total traffic is less than equal to the link capacity. Condition (8.c) guarantees that the flows from the subscribers are not affected by traffic generated by ad hoc buyer. However, the constraint (8.c) makes the problem (8) difficult to solve, or it may not have a solution. To resolve this, problem (8) can be simplified by relaxing the constraint (8.c) and substituting it with a new constraint $U_s(x_s) \ge 0$, for $s \in S$. This way, it allows CNP to be more flexible with the bandwidth allocation. However, problem (8) becomes a non-convex problem, which is an NP-Hard problem [13]. It is because users with real-time traffic is usually modeled using a sigmoidal function, which is non-concave. Notice that with the new constraint, the connection quality of the subscribers may be compromised when network experiences a bottleneck.

Hence, to resolve problem (8), we consider an equivalent problem. Given CNP's objective is to maximize $\sum_{s \in S} h_s + \sum_{\dot{s} \in \dot{S}} \sum_{\bar{s} \in \dot{s}} x_{\bar{s}} \, g_{\bar{s}}$, where h_s denotes the fixed monthly fee paid by subscribers and $g_{\bar{s}}$ can be interpreted as the cost to support ad hoc buyer $\bar{s}$. Since price h_s is a constant, CNP's objective becomes maximizing the aggregation of $\sum_{\bar{s} \in \dot{s}} x_{\bar{s}} \, g_{\bar{s}}$ .Hence, we consider the equivalent problem.

$$\max \sum_{\dot{s} \in \dot{S}} \sum_{\bar{s} \epsilon \dot{s}} U_{\bar{s}}(x_{\bar{s}}) \quad (9)$$

$$s.t. \sum_{\bar{s} \epsilon l} x_{\bar{s}} \le C_l - \sum_{s \in l} x_s,$$

$$over \ x_{\bar{s}} \ge 0,$$

The constraint in problem (9) prevents subscribers' connection quality to be compromised by traffic from ad hoc buyers. Therefore, packets that belong to subscribers receive a higher priority than those of ad hoc buyers during occurrence of network congestion. Problem (9) is similar to problem (3), which also can be resolved using sub-gradient based algorithm. Thus, buyers solve eq. (5) and CNP determines the minimum price to sell on each link l by solving

$$g_l^{t+1} = \left[g_l^t - \sigma^t \left(\left(C_l - \sum_{s \in S} x_s \right) - \sum_{\dot{s} \in l} \sum_{\bar{s} \epsilon \dot{s}} x_{\bar{s}} \right) \right]^+ . \quad (10)$$

The total *minimum price* to sell to ad hoc buyer $\bar{s}$ is $g_{\bar{s}} = \sum_{\bar{s} \in l} g_l^t$.

Proposition 2: If the step size σ in (6) satisfies (10), then the solution converges to the optimal solution of problem (9).

Proof. The same argument from proposition 1 applies. ■

C. Final Resale Price Charged to Ad hoc Buyers

In summary, CNP presents $g_{\bar{s}}$ to reseller $\dot{s}$, who then charges his/her ad hoc buyer $\bar{s}$ final sale price at $\lambda_{\bar{s}} = max(\lambda_{\dot{s}}, g_{\bar{s}})$ for the service, and the buyer pays $\lambda_{\bar{s}}$ amount.

IV. REVENUE SHARING

In this section, we address how the revenue gained from selling bandwidth to ad hoc buyers is shared between the reseller and CNP. Since resellers depend on CNP's infrastructure to provide ad hoc service, we assert that CNP should be given the greater control of how the revenue should be shared. In order to investigate what sharing mechanism may provide incentives for CNP to support ad hoc service, we explore both *cooperative* and *non-cooperative* strategies.

A. Cooperative Versus Non-Cooperative Strategies

Let us start with the *non-cooperative* strategy. In the non-cooperative model, CNP determines and charges reseller $\dot{s}$ with the minimum price to sell $g_{\bar{s}}^t$ for supporting the ad hoc service at time t. Then, reseller sells his/her bandwidth to ad hoc buyer $\bar{s}$ at price $\lambda_{\bar{s}}^t$ and pays $g_{\bar{s}}^t$ to CNP, for $\bar{s} \in \dot{s}$ and $\lambda_{\bar{s}}^t \ge g_{\bar{s}}^t \ge 0$, and reseller keeps the difference between $\lambda_{\bar{s}}^t$ and $g_{\bar{s}}^t$. In all circumstances, the CNP will not know the final sale price $\lambda_{\bar{s}}^t$ to ad hoc users. This leads to a few implications which may not be favorable to CNP.

Firstly, CNP's knowledge of the demand in ad hoc market will simply be based on ad hoc requests fulfilled; this means CNP will not know the *real* total market demand and dollar value of the ad hoc service market. The absence of this knowledge will lead to loss of market oversight and control over pricing over time. Secondly, reseller $\dot{s}$ may be encouraged to buy bandwidth and resell it for profit by maximizing the difference between $\lambda_{\bar{s}}^t$ and $g_{\bar{s}}^t$. This means resellers may aggressively seek to sell their bandwidth, competing directly with CNP to reach the ad hoc users. This in turn leads to market saturation and revenue loss for CNP. In the long run, when ad hoc access becomes easily available, low data usage subscribers who regularly underutilize their data plan may choose to substitute their subscription for ad hoc network service. For these reasons, non-cooperative model does not seem to be advantageous to CNP.

In the *cooperative* model, in addition to CNP setting the minimum selling price $g_{\bar{s}}^t$ to resellers and resellers setting the final sale price $\lambda_{\bar{s}}^t$ to ad hoc buyers at time t, for $\lambda_{\bar{s}}^t \ge g_{\bar{s}}^t$, CNP and resellers share the total revenue received at price $\lambda_{\dot{s}}^t$. In this model, CNP is informed of the final sale price $\lambda_{\bar{s}}^t$ to ad hoc buyers, and its portion of the revenue corresponds to the price. By having visibility of the final sale price to ad hoc buyers, CNP is able to monitor and assess the actual demand and value of the ad hoc service.

Building on this concept, to discourage resellers from selling for profit, we propose a revenue sharing scheme that decreases the reseller's portion of the revenue as more of the monthly bandwidth quota is used (or sold). Therefore, a reseller receives a higher revenue portion if he/she sells when he/she has used very little of his/her monthly bandwidth quota than if he/she sells when he/she has used the majority of the quota. This approach is designed to discourage heavy data usage subscribers from selling their bandwidth, and to allow low data usage subscribers a means to partially offset his/her subscription fee. To fully develop this concept, we propose a revenue sharing mechanism based on Shapley value [16,17]. This mechanism is desirable because it exhibits several fairness properties that ensure revenue sharing is proportional to each party's contribution to the value of the revenue.

B. Desirable Properties

The design of the revenue sharing mechanism should satisfy these following properties. Let $R(.)$ be the revenue function and variable ϕ denotes a vector of *Shapley value*. Let $R(\{\dot{s}, CNP\})$ denote the total revenue from providing ad hoc access to users in $\bar{S} \in \dot{s}$, given amount of bandwidth $x_{\bar{s}}^t$ and price $\lambda_{\bar{s}}^t$ at time t. Thus, $R(\{\dot{s}, CNP\}) = \sum_{\bar{s} \in \dot{s}} x_{\bar{s}}^t \lambda_{\bar{s}}^t$. The Shapley value has the following desirable properties [16,17]:

Property 1 (efficiency): $\phi(\dot{s}) + \phi(CNP) = R(\{\dot{s}, CNP\})$.

The efficiency property requires the revenue shared to equal to the revenue from the service. In other words, the mechanism does not contribute or receive extra revenue.

Property 2 (Symmetry): If $R(\dot{s} \cup CNP) = R(CNP \cup \dot{s})$, then $\phi(\dot{s}) = \phi(CNP)$.

The symmetry property requires that when CNP and reseller $\dot{s}$ each renders the same contribution, both should receive the same portion of the revenue share.

Property 3 (Dummy player): If $\dot{s}$ is a dummy reseller, $R(\dot{s} \cup CNP) = R(CNP)$ and $\phi(\dot{s}) = 0$.

This property assures that when reseller $\dot{s}$ is not contributing, than $\dot{s}$ receives zero share. This situation may occur when the reseller is also an ad hoc buyer, such as when $\dot{s}$ uses his/her own device with data plan to provide ad hoc connection to his/her other devices. Since reseller $\dot{s}$ relies on CNP infrastructure to sell his/her bandwidth, CNP always has a contribution in supporting the ad hoc network service, but not the reseller.

Property 4 (fairness): For any reseller $\dot{s}$ and CNP, the portion of revenue share is proportional to respective contributions to the total revenue gained for the sale. This property addresses the fairness of revenue sharing between any pair of $\langle \dot{s}, \text{CNP} \rangle$.

Property 5 (additivity): Given two systems $(\{\dot{s}, CNP\}, R)$ and $(\{\dot{s}, CNP\}, R')$, we define the system $(\{\dot{s}, CNP\}, R + R')$, by $(R + R')\ (S) = R(Z) + R'(Z)$, $Z \subseteq \{\dot{s}, CNP\}$. $\phi(R + R') = \phi(R) + \phi(R')$.

This property guarantees that if the revenue of an ad hoc service is additive, then the distributed revenue is the sum of the revenue generated for providing the service. In other words, the revenue distribution of service with revenue R will not be affected by service with revenue R'.

C. Shapley Value Methodology

In this section, we will address the revenue sharing implementation between CNP and reseller $\dot{s}$ using Shapley Value.

Definition 1. The *Shapely value* ϕ is defined by

$$\phi_i = \frac{1}{|N|!} \sum_{\pi \in \Pi} \Delta_i\big(R, Z(\pi, i)\big), \qquad \forall i \in N, \tag{11}$$

where $N = \{\dot{s}, CNP\}$, $Z \subseteq$N, and

$$\Delta_i\big(R, Z(\pi, i)\big) = R(\,Z \cup \{i\} - R(Z)\,), \tag{12}$$

where $i \in N$. Remark: Given (N, R), consider a permutation on π on the set N. Members of set N appear to "collect" their revenue according to the ordering π. For each member in N, let Z_π^i be the set of members preceding member i, where $Z_\pi^i \subseteq N$. The marginal contribution of member i according to π is $\Delta_i\big(R, Z(\pi, i)\big) = R\left(Z_\pi^i \cup \{i\} - R(Z_\pi^i)\right)$. Here, the Shapley value can be interpreted as the expected marginal contribution $\Delta_i(R, Z)$, where Z is preceding i in an uniformly distributed random ordering. Since $|N| = 2$ in this model, that is $N = \{CNP, \dot{s}\}$, the Shapley value for $\dot{s}$ and I can be resolved by the following approach

$$\phi_{CNP} = \frac{1}{2} R(\{CNP\}) + \frac{1}{2}\big(R(\{\dot{s}, CNP\}) - R(\{\dot{s}\})\big) \tag{13}$$

and

$$\phi_{\dot{s}} = \frac{1}{2} R(\{\dot{s}\}) + \frac{1}{2}\ \big(R(\{\dot{s}, CNP\}) - R(\{CNP\})\big), \tag{14}$$

for $R(\{\dot{s}, CNP\}) = \phi_{CNP} + \phi_{\dot{s}}$. Here, reseller $\dot{s}$ will receive $\phi_{\dot{s}}$ of revenue from CNP, while CNP will keep the revenue of ϕ_{CNP} amount. Thus, total revenue earned by CNP is $\sum_{s \in S} h_s + \sum_{\dot{s} \in \dot{S}} \phi_I(\dot{s})$. Notice that, since $\dot{S} \subseteq S$, CNP sells the same bandwidth twice and therefore, gain a higher revenue.

Here, $R(\{\dot{s}\})$ and $R(\{CNP\})$ can be interpreted as the revenue that they will gain if they do not collaborate. In a given time t, $R(\{CNP\})$ is determined by solving the following equation

$$R(\{CNP\}) = \sum_{\bar{s} \epsilon \dot{s}} g_{\bar{s}}^t\, x_{\bar{s}}^t. + \omega_t \left(R(\{\dot{s}, CNP\}) - \sum_{\bar{s} \epsilon \dot{s}} g_{\bar{s}}^t\, x_{\bar{s}}^t \right), \tag{15}$$

where weight $\omega_t = \frac{x_{dp} - x_{unusedBW}^t}{x_{dp}}$. Revenue $R(\{CNP\})$ can be interpreted as the price charged by CNP to support ad hoc network access. However, since reseller $\dot{s}$ relies on CNP to provide the network access, when $R(\{\dot{s}\}) = 0$, then $R(\{CNP\}) = R(\{\dot{s}, CNP\})$, which means CNP keeps the entire revenue of $R(\{\dot{s}, CNP\})$. Thus,

$$R(\{CNP\}) = \begin{cases} eq. (17), & R(\{\dot{s}\}) > 0 \\ R(\{\dot{s}, CNP\}), & R(\{\dot{s}\}) = 0 \end{cases}.$$

Next, our proposal determines $R(\{\dot{s}\})$ should be limited to allowing users with underutilized bandwidth to offset some of the data plan subscription fee they pay each month. This approach of reselling unused bandwidth helps to somewhat alleviate the sentiment of wasting money for underutilizing their data plan at the end of the month. Furthermore, $R(\{\dot{s}\})$ is subjected to

$$R(\{\dot{s}\}) \leq \sum_{\bar{s} \epsilon \dot{s}} \big(R(\{\dot{s}, CNP\}) - R(\{CNP\})\big), \tag{16}$$

for $R(\{\dot{s}\}) > 0$. Condition (16) is to assure CNP's share is at least $R(\{CNP\})$. Since the magnitude of $R(\{\dot{s}\})$ influences how much revenue a reseller $\dot{s}$ receives, $R(\{\dot{s}\})$ is capped according to the law of *diminishing returns*, where the contribution to the total revenue diminishes as $R(\{\dot{s}\})$ grows. Thus, $R(\{\dot{s}\})$ is concave and is defined as follows.

$$R(\{\dot{s}\}) = \widehat{\omega}_t \log\left(\alpha \sum_{\bar{s} \epsilon \dot{s}} \big(R(\{\dot{s}, CNP\}) - R(\{CNP\})\big) \right), \tag{17}$$

where α is a positive constant variable decided by CNP, positive weight function $\widehat{\omega}_t = \frac{x_{unusedBW}^t}{x_{dp}}$ at time t. Here, $x_{unusedBW}^t$ denotes the unused bandwidth at time t and x_{dp} is the total of bandwidth included in the monthly data plan (x_{dp} is replenished at the beginning of the billing cycle). Weight

ω_t is an indication of the usage level, i.e. a user with higher ω_t implies a lower data usage and vice versa. Furthermore, ω_t is also used to limit reselling of bandwidth to certain types of subscribers, that is, the low data usage subscribers to help them reduce the excess of data plan underutilization. To reduce traffic from $\dot{s}$, $R(\{\dot{s}\})$ can be extended as follows.

$$R(\{\dot{s}\}) = \widehat{\omega}_t \log\left(\alpha \sum_{\bar{s}\epsilon\dot{s}}\big(R(\{\dot{s}, CNP\}) - R(\{CNP\})\big)\right) - \beta\, x_{\dot{s}}^t,$$

where $\beta \in \{1,0\}$, $\beta = 1$ if $\dot{s}$ is sending data for personal use, otherwise $\beta = 0$. $\beta\, x_{\dot{s}}^t$ can be interpreted as the additional cost for reseller $\dot{s}$ to transmit or download data while providing service at the same time. Also, in order to prevent reseller from making profit after deducting their subscription fee at the end of the billing cycle, the total amount of revenue received by resellers from reselling transactions does not exceed their subscription fee. Thus,

$$R(\{\dot{s}\}) = \begin{cases} 0, & h(\dot{s}) \le \sum_{k=0}^{K} \phi_{\dot{s}}^k \\ eq(17), & Otherwise \end{cases},$$

where K is the number of reselling transactions per month cycle.

Proposition 3: Reseller's $\dot{s}$ share of revenue decreases as $x_{\dot{s}} \to x_{DP(\dot{s})}$.

Proof. We have $x_{\bar{s}} \in x_{\dot{s}}$, for $\forall \bar{s} \in \dot{s}$, that is the amount of bandwidth used by reseller $\dot{s}$ including bandwidth that he/she has sold. Without losing the originality, let $\lambda_{\dot{s}}$ and $g_{\dot{s}}$ be the average price that reseller $\dot{s}$ charges his/her client $\bar{s}$ and the minimum price to sell determined by CNP respectively.

$$\frac{\partial R(\{\dot{s}\})}{\partial x_{\dot{s}}} = -\frac{1}{2x_{DP(\dot{s})}}\log\big(x_{\dot{s}}\,(\lambda_{\bar{s}} - g_{\dot{s}})\big) + \frac{x_{DP(\dot{s})} - x_{\dot{s}}}{2\,x_{DP(\dot{s})}\,x_{\dot{s}}\,(\lambda_{\bar{s}} - g_{\dot{s}})}.$$

Notice that $\frac{x_{DP(\dot{s})} - x_{\dot{s}}}{2\,x_{DP(\dot{s})}\,x_{\dot{s}}\,(\lambda_{\bar{s}} - g_{\dot{s}})} \to 0$, as $x_{\dot{s}} \to x_{DP(\dot{s})}$ and $-\frac{1}{2x_{DP(\dot{s})}}\log\big(x_{\dot{s}}\,(\lambda_{\bar{s}} - g_{\dot{s}})\big) \le 0$. Thus, $\frac{\partial R(\{\dot{s}\})}{\partial x_{\dot{s}}}$ decreases, as $x_{\dot{s}} \to x_{DP(\dot{s})}$, which also implies that $R(\{\dot{s}\})$ decreases when $x_{\dot{s}} \to x_{DP(\dot{s})}$. Thus, $\phi_{\dot{s}}$ in (14) decreases as $x_{\dot{s}} \to x_{DP(\dot{s})}$, which also satisfies property 1. That is, if $R(\{\dot{s}\})$ decreases, then, $\phi_{\dot{s}}$ also decreases. Next, consider a special case that $R(\{\dot{s}\}) = 0$ when $x_{\dot{s}} = x_{DP(\dot{s})}$. Constraint (16) satisfies property 4, such that $\phi_{\dot{s}} = 0$, which means CNP keeps the entire revenue share. Thus, $\phi_{\dot{s}}$ decreases as bandwidth usage $x_{\dot{s}}$ increases.■

Proposition 3 confirms that subscribers with lower data usage receive a higher share of revenue from the reselling transactions than subscribers with higher data usage. In addition, eq. (17) assures that the portion of reselling revenue gained by subscribers diminishes as the amount of bandwidth used/sold increases. Thus, subscribers with higher data usage have less incentive to sell their unused bandwidth to ad hoc buyers. Naturally, resellers may want to maximize his/her Shapley Value in order to maximize the earning. However, it is very difficult for resellers to determine their maximum earning because maximizing Shapley Value is coNP-hard [18]. The solution to Shapley Value maximization problem is discussed in [18].

C. Implementation

In order to ensure appropriate sharing of revenue between CNP and subscribers who resell, CNP keeps track of the total revenue made by resellers and stores the information at CNP's database. This can be done by CNP requiring ad hoc buyers to pay directly and electronically to CNP. Revenue sharing is then computed by CNP and the reseller's portion of the revenue is credited into his/her billing account at the end of each transaction. Detailed mechanics and the security of electronic payment are discussed in [14]. The implementation of this scheme will provide valuable transaction data which can be used for an in-depth and inter-disciplinary research on user behavior in relation to price, network traffic and user utility. As shown in [10], user behavior has a significant impact on network activities such as price, traffic flow, etc.

V. Simulation and Discussion

The objective of our simulation is to understand the behavior of the revenue sharing between CNP and resellers using the Shapley Value model. More specifically, it is to investigate how the difference between the final sale price set by reseller $\lambda_{\dot{s}}^t$ and the minimum sale price $g_{\dot{s}}^t$ set by CNP influences revenue sharing, and whether the outcome is favorable to CNP. Finally, the simulation is conducted to confirm that the scheme achieves the purposes mentioned in the previous section. In order to clearly derive and depict the results, we have to avoid data noise and use the minimum sale price $g_{\dot{s}}^t$ and the final sale price $\lambda_{\dot{s}}^t$ after convergence.

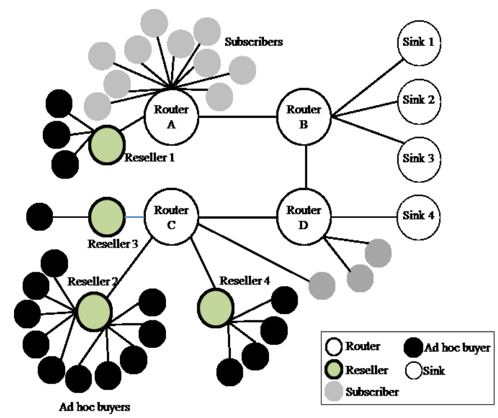

Fig. 2. Network Topology.

A. Simulation Setup

In this simulation, we employ a network with four nodes (node A, B, C, and D) connected by 3 links (link AB, CD, and BD) with a capacity of 50 MB each, as depicted in Figure 2. There are four resellers providing ad hoc service. Each reseller has a maximum of 200 MB in his/her data plan and he/she is hypothetically able to sell all of the bandwidth to ad hoc buyers. In our setup, each reseller sells a maximum of 10 MB per transaction. Reseller *one* is connected to node A and provides service to 3 ad hoc buyers. There are 3 resellers connected to node C: reseller *two* has a buyer, reseller *three* has 10 buyers, and reseller *four* has 4 buyers. There are 4 sinks (one, two, three, and four) that are connected to different ad hoc buyers and subscribers. The path of each connection is described as follows:

- Ad hoc buyers from reseller *one* is connected to *sink one* though link AB.
- Subscribers from router A are connected to *sink two* through link AB.

- Ad hoc buyers of reseller *two* and *three* are connected to *sink four* through link CD.
- Ad hoc buyers of reseller *four* are connected to *sink three* through link CD and DB.
- Subscriber from node C is connected to *sink four* through link CD.
- Subscribers from node D are connected to *sink three* through DB.

The simulation has three parts. First, CNP adjusts the minimum sale price according to the overall bandwidth demand by solving equation (5) and (6). Then, CNP presents a minimum sale price on each link to the resellers in the network: link AB = 30 unit currency, CD = 3 unit currency, and BD = 1 unit currency. Link BD has the highest minimum price because there is a higher level of demand for bandwidth. Simultaneously and independently, resellers compute their price, considering the bandwidth demand from ad hoc buyers by solving equation (5) and (11). The four resellers charge their ad hoc buyers at 31, 26, 5, and 8 unit currencies respectively. Finally, CNP computes the revenue sharing between each reseller and CNP based on the specific revenue earned by each individual reseller.

Prices	*Reseller*			
	1	*2*	*3*	*4*
Reseller's final sale price λ (unit currency)	31	26	5	8
Minimum price to sell g (unit currency)	30	3	3	4

Table 1. Price set by resellers and CNP.

B. Results and Discussion

The results of the revenue sharing between CNP and the 4 resellers are depicted in Figures 3(a) – 3(d). The y-axis is the percentage of revenue apportioned to the reseller and CNP, totaling to 100%. Each unit on the x-axis represents 10 MB of bandwidth unused by reseller, and available for him/her to sell; the higher the number, the more bandwidth he/she still has to sell. For instance, when x = 12, there are still 120 MB available for sale and 80 MB already used.

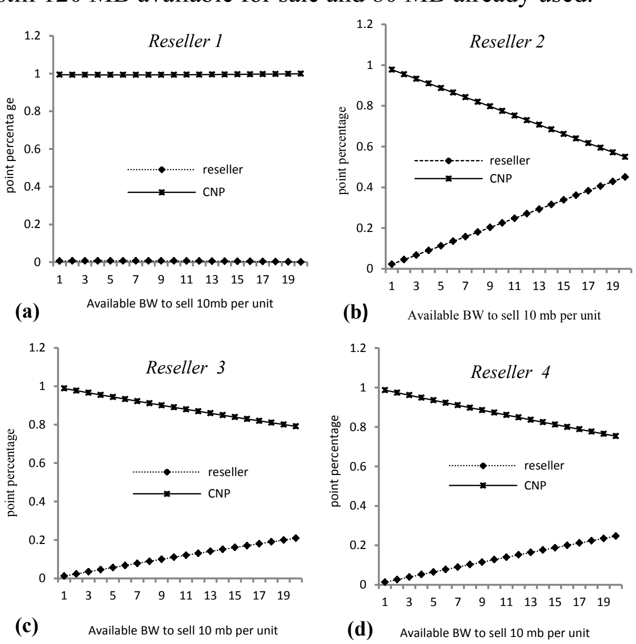

Fig. 3. Revenue sharing between each reseller and CNP according to Shapley value.

We observe several important outcomes. Firstly, from the graphs we can establish that the financial gain is higher for resellers if they sell their bandwidth when they have more unused bandwidth than when they have less. This outcome confirms that the revenue sharing mechanism behaves as intended: it discourages resellers from selling their bandwidth as the amount of bandwidth used increases by lowering their share of the revenue.

Secondly, the simulation outcome of reseller 1 (Figure 3(a)) proves that CNP can discourage bandwidth sale by increasing the minimum price when network experiences high traffic load during peak hours. In this scenario, the high minimum price (30 unit currency) is generated due to a high demand for bandwidth at link AB, including demand from subscribers. Since the demand for ad hoc bandwidth is low, reseller *one* is only able to set his/her price at 1 unit currency higher than the minimum sale price set by CNP (31 unit currency versus 30 unit currency). This means reseller *one* contributes minimally to the transaction, hence receives only a negligible share of the revenue. This strategy also works to encourage resellers to distribute the traffic load to non-peak hours to achieve better revenue share.

Different from reseller *one*, there are many ad hoc users seeking to buy bandwidth from reseller *two*. This leads to the reseller's price to soar to 26 unit currency, while the minimum price for transmitting data on link CD is only set at 5 unit currency by CNP. The case of reseller *two* represents situations where CNP is experiencing low bandwidth demand but reseller is receiving a high level of ad hoc demand. Hence, the minimum price set by CNP is much lower than the reseller's price to ad hoc buyers. In such situations, the reseller has a higher bargaining power because of its higher "contribution" to the transaction. In recognition of this, CNP attributes a relatively higher share of the revenue gained from the transaction to the reseller. However, it is important to highlight here that even in such situations where reseller "contributes" significantly to the revenue gained, our revenue sharing scheme ensures that CNP still gains the majority share of the income, regardless of the amount of bandwidth a reseller has at that point of time, as illustrated. The next simulation confirms this effect in revers: the percentage of the revenue share attributed to CNP increases as the difference between the minimum sale price and final sale price reduces, which characterize the cases with reseller *three* and *four* (Figure 3(c) and 3(d)).

In summary, the simulation confirms that the revenue sharing mechanism is favorable to CNP, as intended. It reduces reseller's portion of the revenue as more bandwidths are used during each subscription cycle, even if the reseller is able to sell the bandwidth at a much higher price than the minimum price set by CNP. Reselling of bandwidth is also discouraged by CNP during peak hours by raising the minimum sale price. The scheme also ensures CNP receives the majority portion of the revenue gained, regardless of amount difference between the minimum sale price and the final sale price. At the same time, the scheme reasonably adjusts resellers' portion of the revenue upward when the minimum price set by CNP is much lower than the reseller's final sale price.

C. Reseller's Maximum Share of Revenue

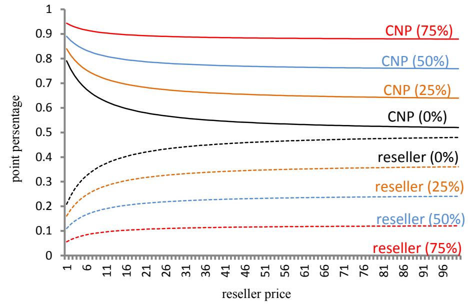

Fig. 4. Maximum and minimum shared received by resellers and CNP.

In the following simulation, we investigate the maximum share of revenue that a reseller may receive at different points of bandwidth usage within a subscription cycle. The simulation is conducted when the reseller's total bandwidth usage is at 0%, 25%, 50%, and 75%. In this simulation, we have a single link network connecting two nodes (node A and B) with a single reseller connected to node A. The reseller sells a total of 10 MB to *two* ad hoc buyers. The total capacity of link AB is 20 MB per unit time. Next, we increase the reseller's price from 0 to 100 unit of currency when bandwidth usage is at 0, 25%, 50%, and 75%. Figure 4(a) illustrates the share of revenue received by the reseller approaches to near 50%, even when bandwidth usage is at 0%. However, the reseller's portion of the revenue subsides as his/her bandwidth usage grows, as illustrated in figure 4.b. Conversely, the lowest portion of revenue CNP receives is at least 50% at all times.

The results from this simulation provide an insight as to when resellers most likely sell their bandwidths. Knowing this and given that CNP knows how many subscribers are at various stages of bandwidth usage in their subscription cycle, CNP is able to anticipate the additional traffic that may be generated from reselling, and devise better management so the connection quality of the subscribers is not compromised. Over time, CNP may be able to estimate the size of ad hoc demand by monitoring the total ad hoc revenue generated by each reseller. Last but not least, it is also important to note that our model of bandwidth reselling and revenue sharing provides CNP with a higher revenue from selling some bandwidth twice: first to subscribers, then to ad hoc buyers through subscribers.

VI. Conclusion

In this paper, we propose a pricing mechanism and revenue sharing scheme based on Shapley value for the redistribution of bandwidth to ad hoc users via subscribers. Contrary to the general perception that provision of ad hoc network access service is disadvantageous to CNP, our pricing and revenue sharing model show that it is possible for CNP to achieve financial gain. We explored the revenue sharing model in two alternative scenarios, cooperative and non-cooperative, and determined that the cooperative model offers better incentives for CNP. In cooperative revenue sharing, CNP will be aware of the reseller's final sale price to ad hoc buyers, and this gives CNP an oversight of the market and more control over pricing. Importantly, our revenue sharing model is able to address critical concerns such as traffic management and dilution of CNP's revenue by discouraging high data usage subscribers from selling and discouraging resale transactions during peak hours. In addition, the scheme also prevents subscribers from selling bandwidth for profit by adjusting the share of revenue apportioned to reselling subscribers according to their level of bandwidth usage and their "contribution" to the final sale price. This model provides incentives for low data usage subscribers to keep their subscription by providing them with an opportunity to earn some income to offset their subscription fee.

In our future work, we will investigate whether the economic interplay and negotiation between CNP, resellers, and ad hoc buyers reach equilibrium. If it does, in what condition equilibrium is reached and how it impacts the revenue sharing mechanism? In addition, in order to design pricing and revenue scheme that reflect quality of ad hoc service, the role QoS in the design will be addressed as well.

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