Localization, Part 1

COMP.4510 and COMP.5490
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Localization

The diagram represents a system dynamics model involving the variables $x_{t-1}$, $x_t$, $x_{t+1}$, $z_{t-1}$, $z_t$, $z_{t+1}$, $u_{t-1}$, $u_t$, and $u_{t+1}$, with the variable $m$ as a control input. The arrows indicate the flow of information or influence between these variables.
Types of maps

**Figure 7.2** Example maps used for robot localization: (a) a manually constructed 2-D metric layout, (b) a graph-like topological map, (c) an occupancy grid map, and (d) an image mosaic of a ceiling. (d) courtesy of Frank Dellaert, Georgia Institute of Technology.
Feature-based maps

- Landmarks used to create topological maps
- Compact representation
- Need a reliable feature detector

Topological mapping

Above: Topological map; Right: DERVISH

Relational graphs for navigation

From Kuipers; also, Kuipers & Byun, Robotics & Autonomous Systems 8: 47–63, 1991
Relational graphs for navigation

Metric: distances, directions, shapes in coordinate system

Topological: connectivity

Landmark definitions, procedural knowledge for traveling

Location-based mapping: Grid map

- Discretize the world into cells
- Grid structure is rigid
- Large maps require substantial memory resources
- Does not rely on a feature detector
- White = unoccupied (0)
  Black = occupied (1)
  Gray = uncertain/unknown (.5)
  - Based on the binary probabilities used for each state
  - We’ll learn how to create grid maps in a few weeks
Discrete Bayes Filter Algorithm

1. Algorithm \texttt{Discrete\_Bayes\_filter}( \textit{Bel}(x),d ):
2. \( \eta = 0 \)
3. If \( d \) is a \textbf{perceptual} data item \( z \) then
4. For all \( x \) do
5. \( \overline{\text{Bel}}(x) = P(z \mid x)\text{Bel}(x) \)
6. \( \eta = \eta + \overline{\text{Bel}}(x) \)
7. For all \( x \) do
8. \( \overline{\text{Bel}}(x) = \eta^{-1}\overline{\text{Bel}}(x) \)
9. Else if \( d \) is an \textbf{action} data item \( u \) then
10. For all \( x \) do
11. \( \overline{\text{Bel}}(x) = \sum_{x'} P(x \mid u,x') \text{Bel}(x') \)
12. Return \( \overline{\text{Bel}}(x) \)
Grid localization algorithm

where $\text{Bel}(x_t) = \{p_{k,t}\}$ where $p_{k,t}$ is defined over a grid cell $x_k$, and $\text{mean}(x_k)$ returns the center of mass of a grid cell $x_k$.
Grid localization

• In a grid map, the world is broken into a metric grid
• In the example above, each grid appears to represent about 1 foot in the x direction
Grid localization example (1)

- When the robot is in an unknown position at the start of localization, all bins of the histogram have an equal probability, with all of the probabilities summing to 1.
Given the sensor readings $p(z|x)$, we can compute $\text{bel}(x)$ by multiplying the sensor value at each grid location with the value from $\text{bel}(x)$, then normalizing the values to sum to 1.

This is the correction step in the Bayes filter algorithm.
\[ p_{k,t} = \eta \ \bar{p}_{k,t} \mid \text{measurement\_model}(z_t, \text{mean}(x_k), m) \]
Now for the prediction step, where we apply the movement model.

In this case, the robot moves 5 feet to the right.

Note that the uncertainty in the movement model causes the probabilities to spread out along x more than the prior step.
\[ \bar{p}_{k,t} = \sum_{i} p_{i,t-1} \text{motion\_model}(\text{mean}(x_k), u_t, \text{mean}(x_i)) \]
Grid localization (4)

- Bayes filter algorithm, correction step:
  - Multiply the state probability distribution by the sensor probability distribution to get the new state probability distribution
\[ p_{k,t} = \eta \tilde{p}_{k,t} \mid \text{measurement\_model}(z_t, \text{mean}(x_k), m) \]
Grid localization (5)

- Again, the robot moves to the right, this time about 10 feet
- Given the uncertainty in the motion model, grid cells become less certain
\[
\bar{p}_{k,t} = \sum_i p_{i,t-1} \text{motion\_model} (\text{mean}(x_k), u_t, \text{mean}(x_i))
\]
Grid localization algorithm

1: Algorithm Grid_localization($\{p_{k,t-1}\}, u_t, z_t, m$):
2:  for all $k$ do
3:      $\bar{p}_{k,t} = \sum_i p_{i,t-1}$ motion_model(mean($x_k$), $u_t$, mean($x_i$))
4:      $p_{k,t} = \eta \bar{p}_{k,t}$ measurement_model($z_t$, mean($x_k$), $m$)
5:  endfor
6: return $\{p_{k,t}\}$

where Bel($x_t$) = $\{p_{k,t}\}$ where $p_{k,t}$ is defined over a grid cell $x_k$, and mean($x_k$) returns the center of mass of a grid cell $x_k$. 
Grid localization demo

• As homework, watch Prof. Fred Martin’s example of grid localization:
  https://www.youtube.com/watch?v=u293629Zwlo
Expanding to more dimensions

Figure 8.2 Example of a fixed-resolution grid over the robot pose variables $x$, $y$, and $\theta$. Each grid cell represents a robot pose in the environment. Different orientations of the robot correspond to different planes in the grid (shown are only three orientations).
Robot Pose

\[ x \]

\[ \theta \]

\[ (x, y) \]
Expanding to more dimensions

Figure 8.2  Example of a fixed-resolution grid over the robot pose variables $x$, $y$, and $\theta$. Each grid cell represents a robot pose in the environment. Different orientations of the robot correspond to different planes in the grid (shown are only three orientations).
Grid localization
Grid localization

Robot position (A)

Robot position (B)

Robot position (C)
Next class

• We’ll be in the lab on Thursday, 5 October
• Will start to implement grid localization in ROS for PS4 in the lab
• Read pages 237-249 of Probabilistic Robotics before class as a review of today’s lecture