

As promised, the substitution model evaluation of  
`((double (double double)) inc) 5)`

First, evaluate `(double double)`:

```
((λ (f) (λ (n) (f (f n)))) (λ (f) (λ (n) (f (f n)))))  
(λ (n) ((λ (f) (λ (n) (f (f n)))) ((λ (f) (λ (n) (f (f n))))) n)))
```

Let's call this `A`.

Now that's passed to `double` in the `(double (double double))` call:

```
(double A)  
((λ (f) (λ (n) (f (f n)))) A)  
(λ (n) (A (A n)))
```

Now this is applied to `inc`: `((double (double double)) inc)`

Let's leave `inc` as its name instead of a lambda, just to simplify things a little.

```
((λ (n) (A (A n))) inc)  
(A (A inc))
```

So let's evaluate `(A inc)`:

```
((λ (n) ((λ (f) (λ (n) (f (f n)))) ((λ (f) (λ (n) (f (f n))))) n))) inc)
```

```
((λ (f) (λ (n) (f (f n)))) ((λ (f) (λ (n) (f (f n)))) inc))
((λ (f) (λ (n) (f (f n)))) (λ (n) (inc (inc n))))

(λ (n) ((λ (n) (inc (inc n))) ((λ (n) (inc (inc n))) n)))
```

Let's call this B.

Now we need to evaluate A applied to this expression:

```
((λ (n) ((λ (f) (λ (n) (f (f n)))) ((λ (f) (λ (n) (f (f n)))) n))) B)
((λ (f) (λ (n) (f (f n)))) ((λ (f) (λ (n) (f (f n)))) B))
((λ (f) (λ (n) (f (f n)))) (λ (n) (B (B n))))
(λ (n) ((λ (n) (B (B n))) ((λ (n) (B (B n))) n)))
```

So we just evaluated ((double (double double)) inc). Now let's apply that to 5.

```
((λ (n) ((λ (n) (B (B n))) ((λ (n) (B (B n))) n))) 5)
((λ (n) (B (B n))) ((λ (n) (B (B n))) 5))
((λ (n) (B (B n))) (B (B 5)))
```

Now we need to figure out (B 5):

```
((λ (n) ((λ (n) (inc (inc n))) ((λ (n) (inc (inc n))) n))) 5)
((λ (n) (inc (inc n))) ((λ (n) (inc (inc n))) 5))
```

`((λ (n) (inc (inc n))) (inc (inc 5)))`

`((λ (n) (inc (inc n))) (inc 6))`

`((λ (n) (inc (inc n))) 7)`

`(inc (inc 7))`

`(inc 8)`

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Now `(B 9)`:

`((λ (n) ((λ (n) (inc (inc n))) ((λ (n) (inc (inc n))) n)) 9)`

`((λ (n) (inc (inc n))) ((λ (n) (inc (inc n))) 9))`

`((λ (n) (inc (inc n))) (inc (inc 9)))`

`((λ (n) (inc (inc n))) (inc 10))`

`((λ (n) (inc (inc n))) 11)`

`(inc (inc 11))`

`(inc 12)`

13

So 13 is the value of `(B (B 5))`. Now we apply `(λ (n) (B (B n)))` to 13

`((λ (n) (B (B n))) 13)`

`(B (B 13))`

Evaluate `(B 13)`:

`((λ (n) ((λ (n) (inc (inc n))) ((λ (n) (inc (inc n))) n)) 13)`

`((λ (n) (inc (inc n))) ((λ (n) (inc (inc n))) 13))`

`((λ (n) (inc (inc n))) (inc (inc 13)))`

`((λ (n) (inc (inc n))) (inc 14))`

`((λ (n) (inc (inc n))) 15)`

`(inc (inc 15))`

`(inc 16)`

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Now `(B 17)`:

`((λ (n) ((λ (n) (inc (inc n))) ((λ (n) (inc (inc n))) n)) 17)`

`((λ (n) (inc (inc n))) ((λ (n) (inc (inc n))) 17))`

`((λ (n) (inc (inc n))) (inc (inc 17)))`

```
((λ (n) (inc (inc n))) (inc 18))
```

```
((λ (n) (inc (inc n))) 19)
```

```
(inc (inc 19))
```

```
(inc 20)
```

```
21
```

This is the answer. Any questions? 😊