As promised, the substitution model evaluation of

(((double (double double)) inc) 5)

First, evaluate (double double):

(((\lambda (f) (\lambda (n) (f (f n)))) (\lambda (f) (\lambda (n) (f (f n)))))
(\lambda (n) ((\lambda (f) (\lambda (n) (f (f n)))) ((\lambda (f) (\lambda (n) (f (f n)))) n)))

Let's call this A.

Now that's passed to double in the (double (double double)) call:

(double A)

(((\lambda (f) (\lambda (n) (f (f n)))) A)
(\lambda (n) (A (A n)))

Now this is applied to inc: ((double (double double)) inc)
Let's leave inc as its name instead of a lambda, just to simplify things a little.

(((\lambda (n) (A (A n))) inc)
(A (A inc))

So let's evaluate (A inc):

(((\lambda (n) ((\lambda (f) (\lambda (n) (f (f n)))) ((\lambda (f) (\lambda (n) (f (f n)))) n))) inc)
(((\lambda (f) (\lambda (n) (f (f n)))) ((\lambda (f) (\lambda (n) (f (f n)))) inc))
  ((\lambda (f) (\lambda (n) (f (f n)))) (\lambda (n) (inc (inc n)))))

(\lambda (n) ((\lambda (n) (inc (inc n))) ((\lambda (n) (inc (inc n)))) n))

Let’s call this B.

Now we need to evaluate A applied to this expression:

(((\lambda (n) ((\lambda (f) (\lambda (n) (f (f n)))) ((\lambda (f) (\lambda (n) (f (f n)))) n)) B)
  ((\lambda (f) (\lambda (n) (f (f n)))) ((\lambda (f) (\lambda (n) (f (f n)))) B))

  ((\lambda (f) (\lambda (n) (f (f n)))) (\lambda (n) (B (B n))))

  (\lambda (n) ((\lambda (n) (B (B n))) ((\lambda (n) (B (B n)))) n)))

So we just evaluated ((double (double double)) inc). Now let’s apply that to 5.

  ((\lambda (n) ((\lambda (n) (B (B n))) ((\lambda (n) (B (B n)))) n)) 5)

  ((\lambda (n) (B (B n))) ((\lambda (n) (B (B n)))) 5))

  ((\lambda (n) (B (B n))) (B (B 5)))

Now we need to figure out (B 5):

  ((\lambda (n) ((\lambda (n) (inc (inc n))) ((\lambda (n) (inc (inc n)))) n)) 5)

  ((\lambda (n) (inc (inc n))) ((\lambda (n) (inc (inc n)))) 5))
((\(n\) (inc (inc n))) (inc (inc 5)))
((\(n\) (inc (inc n))) (inc 6))
((\(n\) (inc (inc n))) 7)
(inc (inc 7))
(inc 8)
9

Now \((B \ 9)\):

((\(n\) ((\(n\) (inc (inc n))) ((\(n\) (inc (inc n))) \(n\))) \(n\)) 9)
((\(n\) (inc (inc n))) ((\(n\) (inc (inc n))) 9))
((\(n\) (inc (inc n))) (inc (inc 9)))
((\(n\) (inc (inc n))) (inc 10))
((\(n\) (inc (inc n))) 11)
(inc (inc 11))
(inc 12)
13

So 13 is the value of \((B \ (B \ 5))\). Now we apply \((\lambda \(n\) \ ((B \ (B \ n))))\) to 13
(\(\lambda\ (n)\ (B\ (B\ n))\)\ 13)

(B\ (B\ 13))

Evaluate (B 13):

\[
(\lambda\ (n)\ (\lambda\ (n)\ (\text{inc}\ (\text{inc}\ n)))\ ((\lambda\ (n)\ (\text{inc}\ (\text{inc}\ n)))\ n))\ 13
\]

\[
(\lambda\ (n)\ (\text{inc}\ (\text{inc}\ n)))\ ((\lambda\ (n)\ (\text{inc}\ (\text{inc}\ n)))\ 13))
\]

\[
(\lambda\ (n)\ (\text{inc}\ (\text{inc}\ n)))\ (\text{inc}\ (\text{inc}\ 13))
\]

\[
(\lambda\ (n)\ (\text{inc}\ (\text{inc}\ n)))\ (\text{inc}\ 14))
\]

\[
(\lambda\ (n)\ (\text{inc}\ (\text{inc}\ n)))\ 15
\]

(inc\ (inc\ 15))

(inc\ 16)

17

Now (B 17):

\[
(\lambda\ (n)\ ((\lambda\ (n)\ (\text{inc}\ (\text{inc}\ n)))\ ((\lambda\ (n)\ (\text{inc}\ (\text{inc}\ n)))\ n))\ 17)
\]

\[
(\lambda\ (n)\ (\text{inc}\ (\text{inc}\ n)))\ ((\lambda\ (n)\ (\text{inc}\ (\text{inc}\ n)))\ 17))
\]

\[
(\lambda\ (n)\ (\text{inc}\ (\text{inc}\ n)))\ (\text{inc}\ (\text{inc}\ 17)))
\]
((\ (n) (inc (inc n))) (inc 18))
((\ (n) (inc (inc n))) 19)
(inc (inc 19))
(inc 20)
21

This is the answer. Any questions? 😊