Modeling Frame-level Errors in GSM Wireless Channels

Ping Ji, Benyuan Liu, Don Towsley and Jim Kurose
Department of Computer Science
University of Massachusetts
Amherst MA 01003
{jiping, benyuan, towsley, kurose}@cs.umass.edu

***Abstract*—We compare four different approaches towards modeling frame-level errors in GSM channels. One of these, the Markov-based Trace Analysis model (MTA), was developed for the purpose of modeling a GSM channel. The next two, k-th-order Markov models and hidden Markov models (HMMs) have been widely used to model loss in wired networks. All three of these have difficulty modeling empirical GSM frame-level error traces. The MTA model and HMM predict frame error rates substantially different from that measured from the trace, and all three models have difficulty capturing the long term temporal correlation structure. We propose a fourth model, the extended On/OFF model, which alternates between an ON (error-free) and an OFF (error-filled) state. The state holding times are taken from mixtures of geometric distributions. We show that this model, with mixtures of three or four geometric distributions captures first order and second order statistics significantly better than the preceding three approaches.**

I. INTRODUCTION

There has been substantial recent interest in the development and deployment of wireless data communication systems such as 3G wireless networks, IEEE 802.11, and Bluetooth. The characteristics of such wireless channels (in particular, with respect to frame-level errors) are quite different from those of traditional wired links [8]. Most wireless channel modeling efforts have focused on physical layer properties such as the signal to noise ratio. Communication protocols at the network layer and above, however, operate on the basic unit of a frame (or packet). Frame-level wireless channel models are thus of particular interest in the design and evaluation of such protocols in wireless networks.

A recent paper by Konrad et al. [5] proposed and evaluated a frame-level error model for GSM channels. They used a high-order Markov model to capture channel behavior in a so-called *bad state*, and empirically fit exponential distributions to measured data to model the lengths of alternating intervals of time spent in "bad" and "good" states. Their Markov-based Trace Analysis (MTA) model is shown to capture the correlation structure during "bad" periods better than the classical Gilbert model [9]. However, the MTA model is unable to match important statistics such as the frame error rate and the autocorrelation function of the frame-level error time series.

In this paper we consider three alternative approaches towards modeling frame-level errors in GSM channels. Our analysis of the frame-level traces in [5] reveals temporal correlation in the frame-level error process over relatively long time scales, as well as high variability in the distribution of consecutive error-filled and error-free frames, neither of which is captured by the model proposed in [5]. The first two approaches, based on k-th order Markov chains and hidden Markov models, have been used to model wired links. Unfortunately, they do little better than the MTA model to accurately model various first order and second order statistics, even when they are allowed to have relatively large state spaces. On the other hand, we find that a two state semi-Markov model (henceforth referred to as an *extended ON/OFF model*), where the state holding times of one or both states are characterized by mixtures of geometric distributions captures first order and second order statistics such as frame error rate and autocorrelation function. Furthermore, we find that no mixture of more than four geometric distributions are needed.

The remainder of the paper is organized as follows. In Section II, we analyze the GSM channel frame-error trace from [5], study its statistical properties, and identify the characteristics of the frame-error trace that should be captured in a frame-level error model. Section III describes the MTA model and two other approaches that have been used to successfully model packet errors in wired links, and their application to modeling GSM channels. In Section IV we describe our extended On/Off model and demonstrate its ability to more accurately capture both first and second order statistics of the frame-error process. Section V concludes the paper.

II. MODELING GSM CHANNELS

We begin by analyzing the GSM channel link-layer frame-error trace gathered by Konrad et al. [5]. In this trace, frame-level wireless communication was recorded for approximately 215 minutes between a laptop host (with a signal strength below 4 on a scale of 1 to 5) and a fixed end host. When a frame was received without bit errors, a '0' was recorded, when a corrupted frame was received, a '1' was recorded (regardless of whether the underlying link layer protocols was able to recover the frame via retransmission [7], [5]). Thus, a binary sequence representing the frame errors of GSM channel was generated; see [5] for details.

Figure 1 shows the frame error rate (FER) of this GSM trace smoothed over a window of 10,000 samples. While the entire trace shows large variations over time, we can roughly identify three distinct portions of the trace, as shown in Figure 1. Part1 and part3 each contain approximately 225,000 samples, while

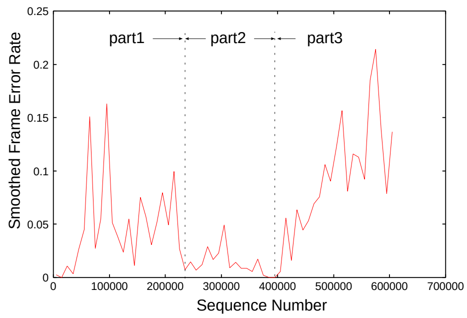

Fig. 1. Smoothed Error Rate of Empirical Data With Window Size 10k

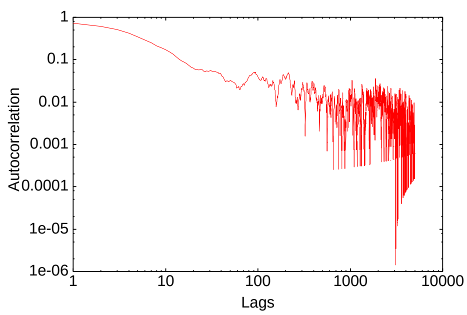

Fig. 2. The Autocorrelation Function of The Second Part of GSM Trace

part2 contains about 170,000 samples. Note that the third part of the trace shows a trend of increasing FER.

Since our goal is to develop Markov models for stationary loss processes, it is important to identify segments of the trace that satisfy a stationarity criteria. We study the stationarity of these three trace segments separately using the runtest and the reverse arrangements test [1], which perform hypothesis tests on the stationarity of random data sequences. In both of these tests, each subtrace is divided into K equal length segments. A sample mean is calculated for segment i, and is used in the stationarity tests. The results of both runtest and reverse arrangements test typically depend on the segment length used, and thus we applied these tests to the entire GSM trace, and to the three subtraces separately, using various segment lengths. We found that, at 0.02 level of significance, the whole GSM trace passes the stationarity tests only when the segment size is larger than 50,000 samples. Part1 passes the stationarity tests when the segment size exceeds 2,200 samples, part2 passes the stationarity tests with a segment size larger than 800 samples, and part3 cannot pass the reverse arrangements test because of its significant increasing trend. Given these observations, we chose to study the three trace parts separately. We begin with part2, which is most likely to be stationary (see details in [4]).

We are interested in modeling frame-error statistics of GSM channels such as the loss rate and the autocorrelation function. We first examine these statistics in the empirical trace. Figure 2 shows the autocorrelation function of the second part of the GSM frame-error trace; part1 and part3 exhibites 'higher correlation. We next consider the measured lengths of consecutive corrupted and error-free frames. Let us define an error burst as a sequence of consecutive corrupted frames (1's), and an error-free burst as a sequence of consecutive correct frames (0's). Let $\bar{X}$ be the average error burst length, let $\bar{Y}$ be the average error-free burst length, and let σ_X and σ_Y be the standard deviation of error and error-free burst length, respectively. The coefficient of variation (Cov) of error and error-free burst lengths are defined as

$$Cov(X) = \sigma_X / \bar{X} \quad (1)$$
$$Cov(Y) = \sigma_Y / \bar{Y} \quad (2)$$

The calculated values of Cov of the error and error-free burst lengths are show in Table I. From Table I, we observe that the $Cov(X)$ and $Cov(Y)$ are greater than 1, implying that the burst lengths demonstrate higher variability than that of a geometric distribution (which has a coefficient of variation less than or equal to one).

	CoV of Burst Length	
	error burst	error-free burst
part1	2.064	5.116
part2	1.087	3.452
part3	2.850	5.875

TABLE I

COEFFICIENT OF VARIATION VALUES FOR ERROR AND ERROR-FREE BURSTS

Having now considered selected performance measures from the trace data, let us next consider the performance measures that we want to match with a model (i.e., those measures that we want to be able to derive from a model and that matches well with the empirically-observed trace data).

We focus on the following first order and second order statistics in our analysis. Let $\{Z_i\}$ denote a wide sense stationary sequence of random variables corresponding to the frame error trace. Let $\{X_i\}$, $\{Y_i\}$ denote the lengths of error- and error-free bursts, respectively.

- *Frame error rate (FER).* The frame error rate is given by:

$$FER = \frac{\bar{X}}{\bar{X} + \bar{Y}} \quad (3)$$

- *Complementary Cumulative Distribution Function (CCDF).* The CCDF of X and Y are defined as

$$F_X^{(c)}(x) = P\{X > x\} \quad (4)$$
$$F_Y^{(c)}(y) = P\{Y > y\} \quad (5)$$

- *Autocorrelation.* The autocorrelation of a frame-error trace is defined as

$$\rho_Z(h) = E[(Z_{i+h} - \bar{Z})(Z_i - \bar{Z})]/E[(Z_i - \bar{Z})^2] \quad (6)$$

where, h is the lag, and $\bar{Z}$ is the sample mean of Z.

III. THREE MODELS FOR PREDICTING FRAME-ERROR PERFORMANCE MEASURES

As noted in the Introduction, a recent paper by Konrad et al. [5] proposed and evaluated a model for frame-level errors in

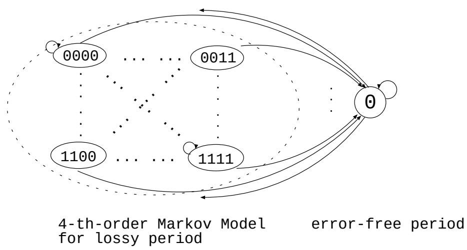

Fig. 3. Converting the MTA algorithm to a k-th-order MTA Model (here, $k = 4$)

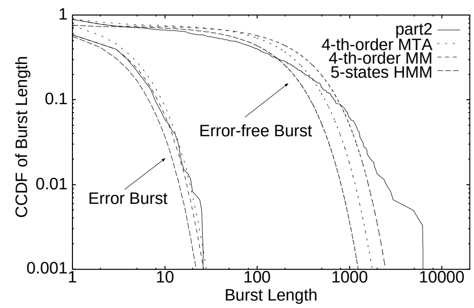

Fig. 4. Comparison of Complementary Cumulative Distribution for Error- and Error-free Burst Length of Existing Related Models

GSM channels. In this section, we review that model, as well as two other modeling approaches - k-th-order Markov models and hidden Markov models - that have been used to successfully model loss behavior in wired networks. Our goal here is to determine how well these models predict the first- and second-order frame-error statistics discussed in the previous section. Let us briefly describe these models. We begin with the k-th-order Markov model as it is a component of the model proposed by Konrad et al.

1) k-th-order Markov Model: k-th-order Markov models have been used to characterize packet loss in the Internet [13]. Since the states of a k-th-order Markov model are observable, the transition probability matrix is obtained by directly counting the frequencies of transitions occurring in the sample traces. Note that when $k = 1$, the k-th-order Markov model reduces to the two-state Markov model. Since a k-th-order Markov model needs 2^k states, the value for k is usually chosen to be small (typical values for k are 2 to 6).

2) MTA algorithm: In [5], Konrad et al. propose a *Markov-based Trace Analysis* (MTA) algorithm. The MTA algorithm divides the frame error trace into consecutive *lossy periods* and *loss-free periods*. A lossy period starts with an error frame, and terminates after a consecutive number (C) of correctly received frames. Here, C is the sum of the mean and the standard deviation of error-frame burst lengths. After the trace is divided into lossy and loss-free frame sequences using C, an exponential distribution is used to fit the sequence length for each set. While in an loss-free period, the output can only be '0' (the frame contains no error). In a lossy period, the output is determined by the state of a k-th-order Markov model. The concatenated sequence of all the lossy period sequences is used to calculate a k-th-order Markov model. To generate synthetic traces of correct or corrupted frames, the MTA algorithm first determines the lengths of a pair of error-free and lossy states according to these two exponential distributions. Frame sequences are then generated as follows - a sequence of 0s fills in the error-free period, and a sequence of 0s and 1s, generated by the k-th-order Markov model, is used to fill in the lossy period.

A careful study of the MTA algorithm reveals that it is essentially equivalent to the discrete-time Markov model illustrated in Fig. 3. In Fig. 3, a separate state can be used to represent the error-free state, where the transition probabilities for leaving or entering this special state are derived from the two exponential fitting parameters. Also, a k-th-order Markov model is used to model the concatenated lossy periods. In Fig. 3, we show an example of the MTA model using a 4-th-order Markov model to model the lossy period. The transformation procedure and a proof of equivalence can be found in [4]. In this paper, for the convenience of comparison, we will refer to the converted MTA algorithm as the k-th-order MTA model, where k denotes the order of a high-order Markov model that is used to model the lossy period.

3) Hidden Markov Models: Hidden Markov Models (HMM) [10], have recently been used to characterize Internet link losses [11], and have been found to better match certain statistics of the loss process than k-th-order Markov models with a larger number of states. In a HMM, the states and transitions between states are not observable. Each state can produce an output when a transition is made, with the distribution of produced outputs being state-dependent. For example, to model the frame error traces, the possible outputs would be 0 and 1. The EM algorithm, also known as the *Baum-Welch* algorithm, is used to estimate the parameters of the HMM that maximize the probability of outputting the observed sequence. While HMMs are generally effective, the time needed to estimate their parameters can be very long, and the EM algorithm can become trapped in locally maximum solutions.

Let us now consider how well the performance measures predicted by the models match the measures found in the trace data. Two approaches are possible here: *(i)* use a model to generate synthetic traces and evaluate the measures as found in these traces, or *(ii)* for our Markovian models, derive the performance measures directly from the model. For example, for a Markov model, we can use the state stationary distribution π to calculate the frame error rate as

$$FER = \sum_{i=1}^{N} \pi_i \cdot b_i(1) \qquad (7)$$

where N is the number of states, and $b_i(1)$ is the probability that state i generates output symbol 1. For a hidden state, $0 \le b_i(1) \le 1$, and for an observable state, $b_i(1)$ is either 1 or 0. We'll use this second approach here. For detailed definitions and calculations of each of the performance metrics derived from the models, see [4] and [12], [6], [10], [11].

Figure 4 plots the burst length distributions for all three models, as well as for the empirical data from part2 of the GSM trace; Figure 5 plots their autocorrelations. We observe that the 4-th-order MTA model, the 4-th-order Markov model, and the

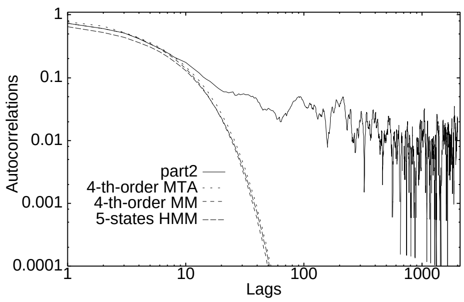

Fig. 5. Comparison of Autocorrelation of Existing Related Models

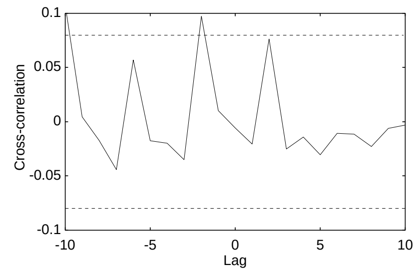

Fig. 6. Cross-correlation between error- and error-free burst length of the GSM frame error trace

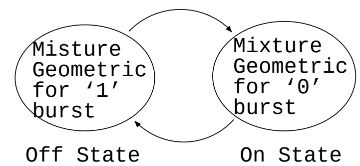

Fig. 7. An Extended On/Off Model

5-state HMM all provide error-burst length distribution reasonably close to that of the original trace. However with respect to the error-free burst length distribution, these models differ significantly from the original trace in their tail behavior. We also observe that these models do not capture the autocorrelation well, as shown in Fig. 5. Finally, we note that as shown in Table II of Section IV, the 4th-order MTA model and the 5-state HMM do not even predict the frame error rate accurately.

These results suggest that existing models do not capture the tail behavior of error-free burst length, the autocorrelation, or even the frame error rate very well. In addition, in [4], we also show that for part1 and part3, these three models do not capture the tail behavior of the error burst length distribution either. In the next section, we use insights gained from our trace analysis to develop an extended On/Off model with a structure that constrains transitions among states in the On-period and Off-period in order to more accurately reflect the variations in burst length.

IV. The Extended On/Off Model and its Evaluation

Recall from Section II our analysis of the empirical traces indicates that the error- and error-free burst-length distributions exhibit a greater variability than the geometric distribution. Furthermore, the trace exhibits significant temporal correlation. On the other hand, our analysis indicates that successive loss and loss-free burst lengths are independent. Let $\rho(h)$ be the sample correlation between the error- and error-free burst-length, with a lag h. We have

$$\rho(h) = \frac{\frac{1}{n-h}\sum_{i=1}^{n-h}(X_i - \bar{X})(Y_{i+h} - \bar{Y})}{\sigma_X \sigma_Y} \qquad (8)$$

where, n is the number of sample bursts, $\{X_i\}$ and $\{Y_i\}$ are i.i.d. sequences of random variables corresponding to error- and error-free burst lengths, with averages $\bar{X}$ and $\bar{Y}$, and standard deviations σ_X and σ_Y, respectively. In Fig 6, we show the cross-correlation, ρ, between error- and error-free burst length of part2. The confidence bounds of ρ with significance level 0.05 are indicated as dashed lines in Fig. 6. [1] From Fig. 6, we observe that the cross-correlation of error and error-free burst-length is negligible for the second part of the GSM frame error trace. This suggests that the frame-level error process might be well modeled by a discrete time two state model in which frames are lost while the model is in the OFF (error) state and not lost while the model is in the ON (error-free) state. In order to capture the variability in the amount of time spent in each of these states, a mixture of geometric distributions is used. To avoid confusion, we will henceforth refer to the geometric distributions as geometric phases.

[1] For computation details of confidence bounds of cross-correlations, see [2].

To summarize, the state holding times of our extended On/Off model of the GSM channel are characterized by mixtures of geometric phases. The mixtures for each state are determined using the EM algorithm. In order to initialize this algorithm, we use the procedure introduced in [3]. Details of the fitting algorithm can be found in [4].

As in the previous section, we are interested in how well the performance measures predicted by the extended On/Off model match the measured observations in the empirical traces, and how the predictions of the extended On/Off model compare with those of the other models we encountered in the previous section.

In Table II, we compare the predicted frame-error rate of all the models presented in this paper. For HMM, with different initial estimates of the parameters, the final estimated HMMs could also be different [10]. Thus, we computed ten 5-state HMMs, and their corresponding FERs, and show the mean value and the errors of the FERs in Table II.

	Frame Error Rate
GSM trace	0.0134
4th order MTA	0.0209
4th order MM	0.0139
5-state HMM	0.014 ± 0.006
Extended On/Off	0.0134

TABLE II

Prediction of Frame Error Rate for part2.

From Table II, we observe that the Extended On/Off model (using one geometric phase in the Off-period and three geometric phases in the On-period) most accurately captures the FER for part2 of the GSM trace. Other models, even with larger

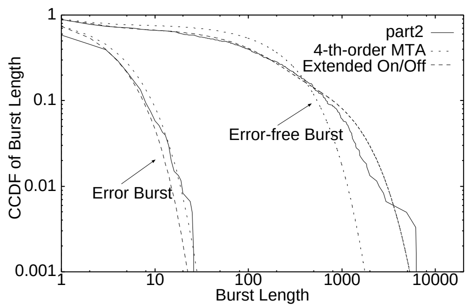

Fig. 8. Comparison of Complement Cumulative Distributions for Error and Error-free Burst Length provided by the 4-th-order MTA Model and the Extended On/Off Model

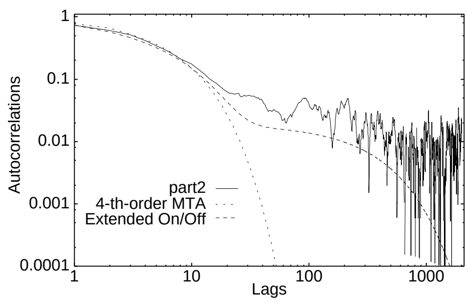

Fig. 9. Comparison of Autocorrelations Provided by the 4-th-order MTA Model and the Extended On/Off Model

state spaces, fare less well. Note that although the HMM considered in Table II has only 5 states, the complexity of computing its parameters is significantly greater than that of the extended On/Off model, since transitions and outputs in the latter are much more constrained.

We also computed the CCDF and the autocorrelation of the extended On/Off model, and compared these with those of the 4-th-order MTA model. From our earlier Figures 4 and 5, we know that the 4-th-order MTA model, 4-th-order Markov model, and the 5-state HMM result in similar burst length distributions and autocorrelations. Thus, we only compare the extended On/Off model with the 4-th-order Markov model in Figures 8 and 9. In Figure 8, we show the CCDFs of error- and error-free burst lengths derived from the 4-th-order MTA model and the extended On/Off model, and compare them with the CCDF of the empirical trace. From Figure 8, we observe that the extended On/Off model captures the tail distribution of error-free burst length significantly better than the 4-th-order MTA model. Figure 9 shows the autocorrelations derived from these two models, and the autocorrelation of the empirical trace. From Figure 9, we observe that the extended On/Off model also predicts the autocorrelation much more accurately than the 4-th-order MTA model (or other models with similar performance).

Finally, we also evaluated all of the models using part1 and part3 of the GSM trace data. For those two traces, we again find that the extended On/Off model (using 3 geometric phases to chapture the Off-period and 3-4 geometric phases to chapture the On-period) provides a significantly closer match to the empirical data than the other modeling approaches. Moreover, the degree to which the extended On/Off model outperforms these other models is even greater than in the case of part2 considered here. We conjecture that this may be due to the larger variations in the part1 and part3 traces (see Table I). In addition, we also investigated the performance of extended On/Off model by varying the number of geometric phases that are used to capture the On or Off period. Our results show that the performance of the extended On/Off model is not significantly improved by using more numbers of geometric phases. For details, see [4].

V. Conclusion

In this paper, we have presented four different approaches towards modeling frame-level errors in GSM channels, including a new approach known as the extended On/Off model. We evaluated these approaches against empirical GSM frame-level error traces and found that the extended On/Off model, with a small state space, captures both first order and second order statistics significantly better than previously proposed approaches. Our future work will include the development and evaluation of models for frame errors seen by mobile users.

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References

[1] Julius S. Bendat and Allan G. Piersol. *Random Data: analysis and measurement procedures*. John Wiley & Sons, 1986.

[2] Paul Cohen. *Empirical Methods for Artificial Intelligence*. The MIT Press, 1995.

[3] Anja Feldmann and Ward Whitt. Fitting mixtures of exponentials to long-tail distributions to analyze network performance models. In *INFOCOM (3)*, pages 1096–1104, 1997.

[4] Ping Ji, Benyuan Liu, Don Towsley, Zihui Ge, and Jim Kurose. Modeling gsm wireless channels. Technical Report 02-12, University of Massachusetts at Amherst, 2002.

[5] Almudena Konrad, Ben Y. Zhao, Anthony D. Joseph, and Reiner Ludwig. A markov-based channel model algorithm for wireless networks. In *Fourth ACM International Workshop on Modeling, Analysis and Simulation of Wireless and Mobile Systems (MSWiM)*, 2001.

[6] Rosa M.M. Leao, Edmundo de Souza e Silva, and Sidney C. de Lucena. A set of tools for traffic modeling, analysis and experimentation. *Computer Performance Evaluation / TOOLS*, Vol.1786, 2000.

[7] M. Mouly and M.B. Pautet. *The GSM System for Mobile Communications*. Telecom Publishing, June 1992.

[8] Theodore S. Rappaport. *Wireless Communications: Principle and Practice*. Prentice Hall, 1996.

[9] Sheldon Ross. *Stochastic Processes*. John Wiley and Sons, 1996.

[10] Lawrence R.Rabiner. A tutorial on hidden markov models and selected applications in speech recognition. *IEEE*, Vol.77(2), Feb 1989.

[11] Kavé Salamatian and Sandrine Vaton. Hidden markov modeling for network communication channels. In *ACM Sigmetrics*, 2001.

[12] William Turin and Robert van Nobelen. Hidden markov modeling of flat fading channels. *IEEE Journal on Selected Areas in Communications*, Vol.16(9), December 1998.

[13] Maya Yajnik, Sue Moon, Jim Kurose, and Don Towsley. Measurement and modelling of the temporal dependence in packet loss. In *IEEE Infocom*, New York, NY, 1999.