Probability

\[ x_i = \text{result of } i\text{-th coin flip} \quad x_i = \{H, T\} \]

\[ P(x_1 = x_2 = x_3 = x_4) = \boxed{0.125} \quad P_i(H) = \frac{1}{2} \quad A_i \]

\[ \Rightarrow \frac{1}{16} + \frac{1}{16} = \frac{1}{8} \]

\[ P(\{x_1 x_2 x_3 x_4\} \text{ contains } 3 \text{ } H) = \boxed{1} \]
Probability

\[ P(A) = p \Rightarrow P(\neg A) = 1 - p \]

Independence:

\[ X \perp Y : p(X) p(Y) = p(X, Y) \]

\[ \text{marginals} \]
\[ \text{joint probability} \]
DEPENDENCE

\[ P(X_1 = H) = \frac{1}{2} \]

\[ H: P(X_2 = H \mid X_1 = H) = 0.9 \]

\[ T: P(X_2 = T \mid X_1 = T) = 0.8 \]

\[ P(X_2 = H) = \square \]
Lessons

\[ P(Y) = \sum_i P(Y \mid X=i) \cdot P(X=i) \]

*Total probability*

\[ P(\neg X \mid Y) = 1 - P(X \mid Y) \]

\[ P(X \mid \neg Y) = 1 - P(X \mid Y) \]
Quit

\[ \Pr(D_1) \]

\[ \Pr(D_1 = \text{sunny}) = 0.9 \]

\[ \Pr(D_2 = \text{sunny} | D_1 = \text{sunny}) = 0.8 \]

\[ \Pr(D_2 = \text{rainy} | D_1 = \text{sunny}) = \Box \]
Quit

\[ P(D_1) \]
\[ P(D_1 = \text{sunny}) = 0.9 \]
\[ P(D_2 = \text{sunny} \mid D_1 = \text{sunny}) = 0.8 \]
\[ P(D_2 = \text{rainy} \mid D_1 = \text{sunny}) = 0.2 \]
\[ P(D_2 = \text{sunny} \mid D_1 = \text{rainy}) = 0.6 \]
\[ P(D_2 = \text{rainy} \mid D_1 = \text{rainy}) = \]
Quit

\[ P(D_1) \quad P(D_1 = \text{sunny}) = 0.9 \]
\[ P(D_2 = \text{sunny} \mid D_1 = \text{sunny}) = 0.8 \]
\[ P(D_2 = \text{rainy} \mid D_1 = \text{sunny}) = 0.2 \]
\[ P(D_2 = \text{sunny} \mid D_1 = \text{rainy}) = 0.6 \]
\[ P(D_2 = \text{rainy} \mid D_1 = \text{rainy}) = 0.4 \]

\[ P(D_2 = \text{sunny}) = \_ \_ \_ \_ \_ \]
\[
\begin{align*}
P(D_1) & \quad P(D_1=\text{sunny}) = 0.9 \\
P(D_2=\text{sunny} \mid D_1=\text{sunny}) & = 0.8 \\
\phantom{P(D_2=\text{sunny} \mid D_1=\text{sunny})} & = \boxed{0.2} \\
P(D_2=\text{rainy} \mid D_1=\text{sunny}) & = 0.6 \\
P(D_2=\text{rainy} \mid D_1=\text{rainy}) & = \boxed{0.4} \\
\end{align*}
\]

\[
P(D_2=\text{sunny}) = \boxed{\phantom{0.2}} \quad P(D_2=\text{sunny}) = \boxed{\phantom{0.2}}
\]
Cancer

\[ P(C1) = 0.01 \]
\[ P(C1) = \square \]
Cancer

\[ P(C) = 0.01 \quad P(+|C) = 0.9 \]
\[ P(-|C) = 0.99 \]

\[ P(-|\neg C) = [\text{BLANK}] \]
Cancer

\[ P(C) = 0.01 \quad P(+|C) = 0.9 \]
\[ P(\neg C) = 0.99 \quad P(-|C) = 0.1 \]
\[ P(+|\neg C) = 0.2 \]
\[ P(-|\neg C) = 0.8 \]

\[ P(C|+) = \]

Joint probabilities

\[ P(+, C) = \]
\[ P(-, C) = \]
\[ P(+, \neg C) = \]
\[ P(-, \neg C) = \]
Cancer

\[ P(C) = 0.01 \quad P(+ | C) = 0.9 \]
\[ P(- | C) = 0.1 \]
\[ P(+ | \neg C) = 0.2 \]
\[ P(- | \neg C) = 0.8 \]

Joint probabilities

\[ P(+, C) = 0.009 \]
\[ P(+, \neg C) = 0.001 \]
\[ P(-, C) = 0.188 \]
\[ P(-, \neg C) = 0.792 \]
Cancer

\[ P(C) = 0.01 \quad P(\neg C) = 0.9 \]
\[ P(+ | C) = 0.93 \quad P(\neg + | C) = 0.1 \]
\[ P(\neg + | \neg C) = 0.2 \quad P(+ | \neg C) = 0.8 \]

\[ P(C | +) = \blacksquare \]

Joint probabilities

\[ P(+, C) = 0.009 \]
\[ P(-, C) = 0.001 \]
\[ P(+, \neg C) = 0.188 \]
\[ P(-, \neg C) = 0.792 \]
Bayes Rule

Diagram:

- **A**: not observable
- **B**: observable

Bayesian Network

Diagnosis Reasoning:

- $P(A)$
- $P(B|A)$
- $P(B|\neg A)$
- $P(A|B)$
- $P(A|\neg B)$
Bayes Rule

\[ P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} \]

\[ P(B) = \sum_{A} P(B|A=a) \cdot P(A=a) \]

Total Probability

\[ P(C(+)) = \frac{P(+|C) \cdot P(C)}{P(+)} = \frac{0.9 \cdot 0.01}{0.9 \cdot 0.01 + 0.2 \cdot 0.99} \]